

## School of Mathematical and Computing Sciences Te Kura Pangarau, Rorohiko



# --- Einstein's Gravity --Black holes and Big Bangs...



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Wellington Library
9 September 2009





#### **Abstract:**



Why has the theoretical physics community collectively been racking its brains for the last ninety years trying to understand the implications of Einstein's theory of gravity?

And why have they spent the last fifty years trying to add quantum physics to the mix?

Why should we be interested?

(And how would this affect Astronomy and Cosmology?)

I'll present a non-technical description of some of the key issues...





What on earth is going on?

Why are we interested?

Let's look at the possibilities...





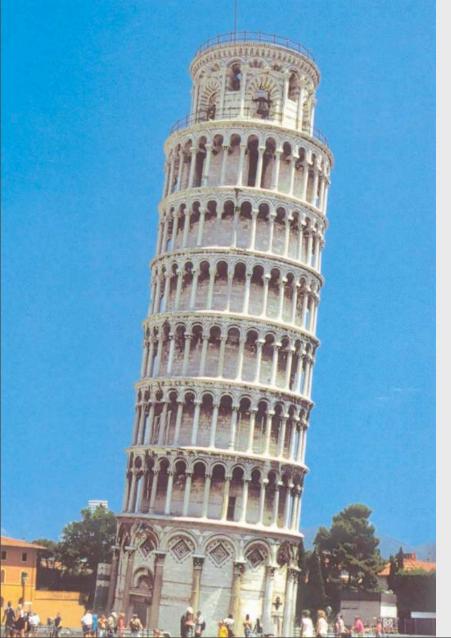


The first really quantitative results on gravity were due to Galileo...

If we neglect air resistance, and this is sometimes a good approximation, then gravity is \*\*\*universal\*\*\*







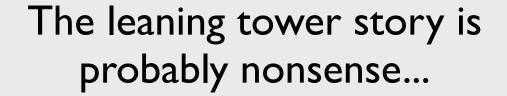
Near the surface of the earth, the acceleration due to gravity is universal and approximately:

g ~ 9.81 metres/sec/sec

g ~ 32 feet/sec/sec



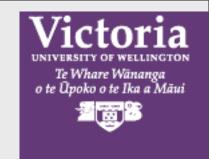


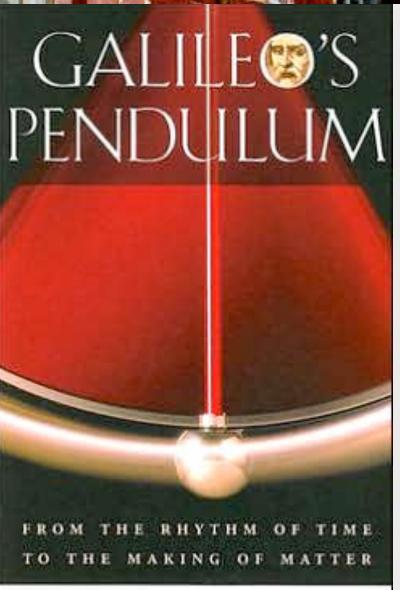


The pendulum story is however, definitely historical fact...

(Something to do when the sermon is in full swing...)



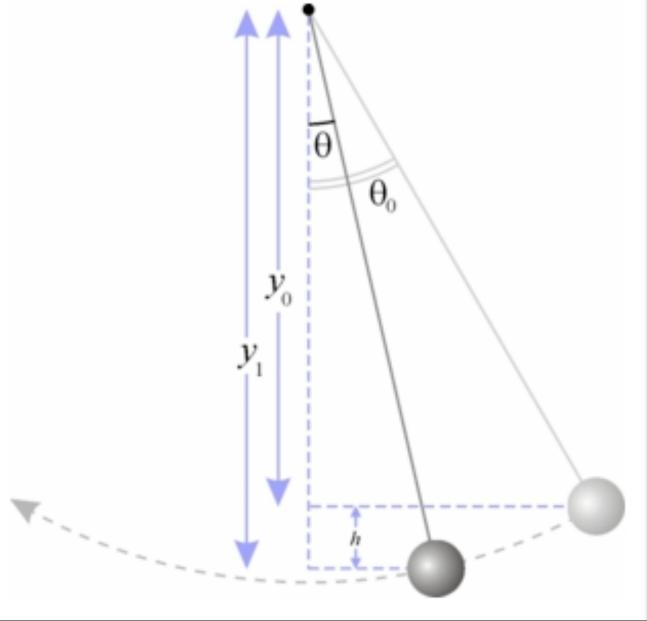




By timing the pendulum, the chandelier of the cathedral at Pisa, and noting that the time for a single beat was independent of how much it was swinging, Galileo could "see" the beginnings of mathematical physics...







This inspiration then led Galileo to carry out some experiments...



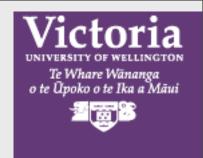


#### Galileo's experiments:

- Pendulums nearly return to their release heights.
- All pendulums eventually come to rest, with the lighter ones coming to rest faster.
- The period is independent of the bob weight.
- The period is independent of the amplitude.
- The square of the period varies directly with the length.

The period is independent of the bob material...



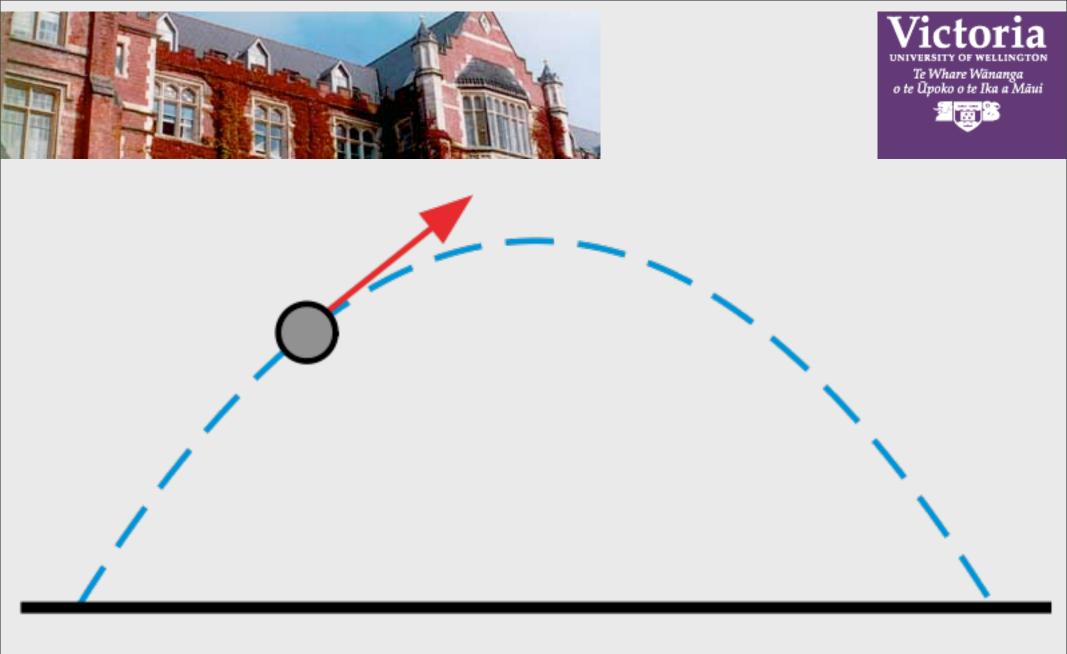


Technologically, this led to the pendulum clock, eventually to the "grandfather clock"...

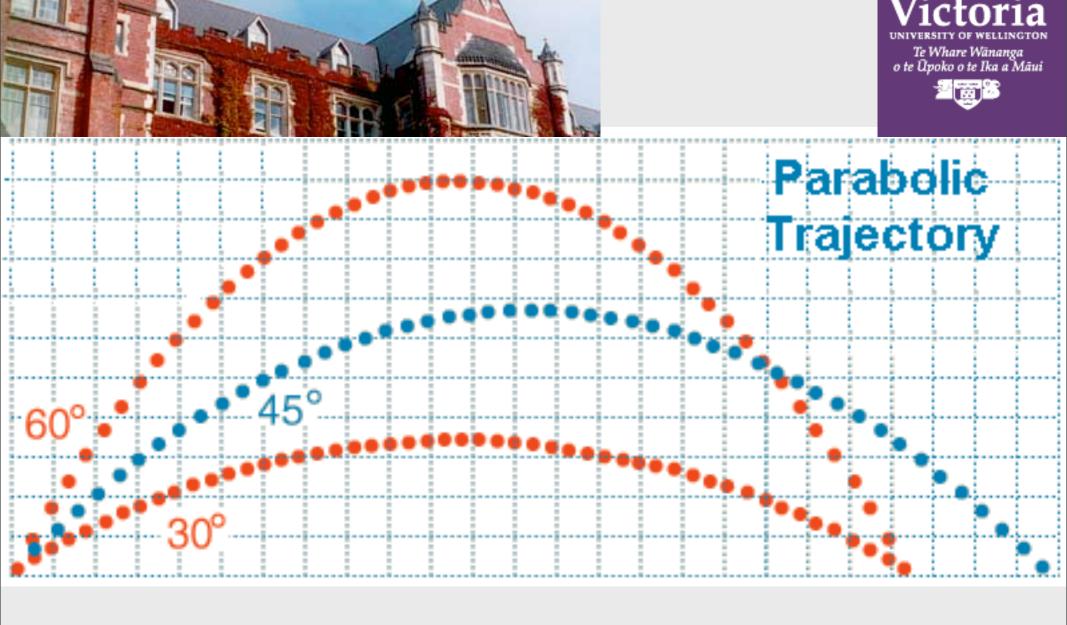
Physically, this led to the notion of gravity as a force of universal acceleration...

(at least near the surface of the earth)

Mathematically, it led to all those "constant acceleration" problems... (ask your teenage offspring, provided they are taking calculus and/or physics)...



Neglecting air resistance is a reasonably good approximation for cannon balls



If it is safe to neglect air resistance, then 45 degrees gives you maximum range...





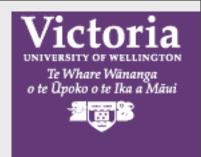


Next came Newton

Inverse-square law

Gravity near the surface of the earth is intimately related to the orbit of the moon around the earth...





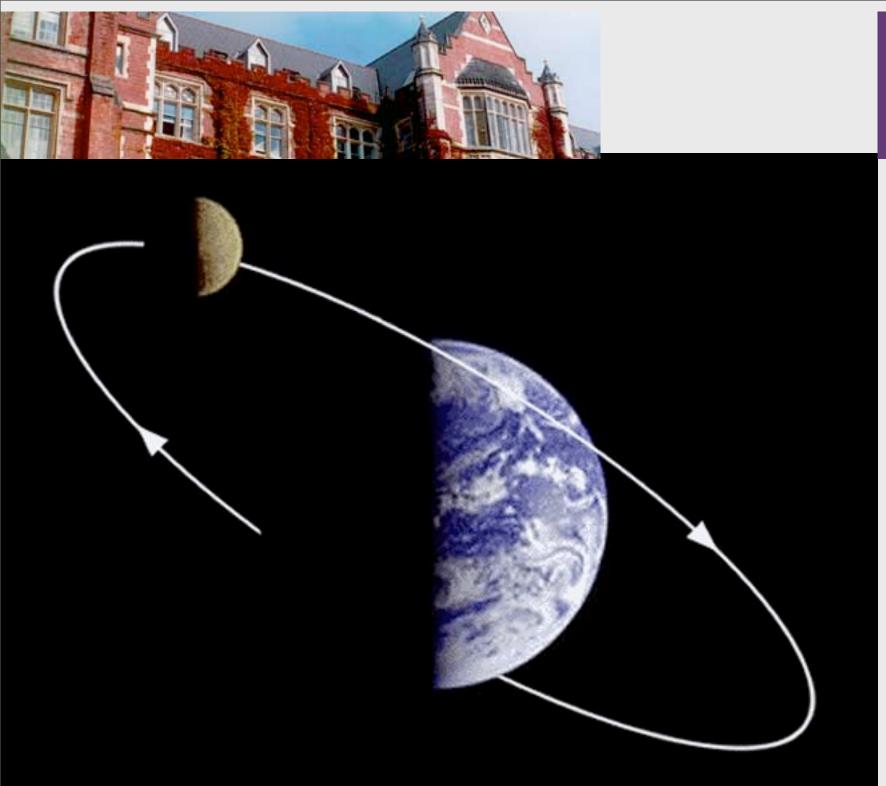


"Nothing yet. ... How about you, Newton?"

#### Inverse-square law

The acceleration due to gravity falls off as the square of the distance...

$$g(r) = g_S \left(\frac{r_S}{r}\right)^2$$



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#### Prediction based on inverse square law:

$$g(r) = g_S \left(\frac{r_S}{r}\right)^2$$

Radius of earth = 6371 km

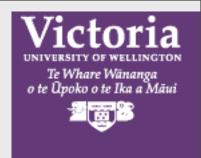
Radius of moon's orbit = 384748 km

acceleration due to gravity at earth's surface = 9.81 metres/sec/sec

Predicted acceleration of moon =  $(9.81 \text{ metres/sec/sec}) \times \left(\frac{6.371 \text{ km}}{384.748 \text{ km}}\right)^2$ 

Predicted acceleration of moon = 0.00269 metres/sec/sec





#### Observation:

Period of moon's orbit =  $27.321 \text{ days} = 2.351 \times 10^6 \text{ sec}$ 

Speed of moon in its orbit = 1.023 km/sec = 1023 metres/sec

Observed acceleration of moon 
$$=\frac{v^2}{R} = \frac{(1023 \text{ metres/sec})^2}{384 748 000 \text{ km}}$$

Observed acceleration of moon = 0.00272 metres/sec/sec





#### Comparison:

Observed acceleration of moon = 0.00272 metres/sec/sec

#### versus

Predicted acceleration of moon = 0.00269 metres/sec/sec

1% agreement...

(Not bad for a very crude approximation using circular orbits...)



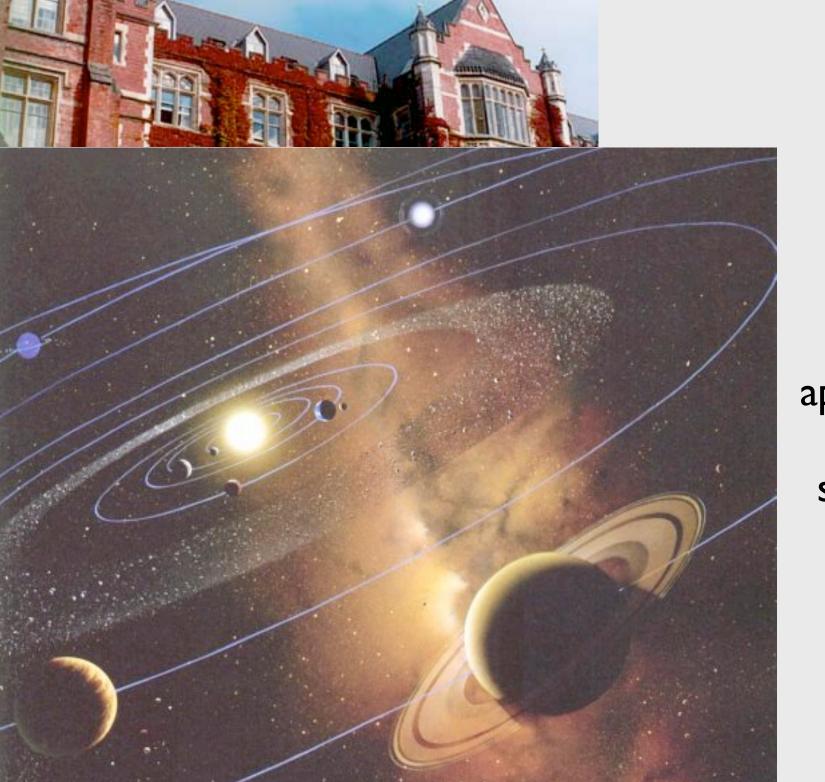


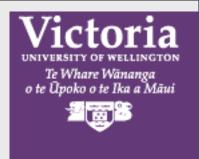
## "As above, so below"

"As below, so above"

The key point is that Newton realized that gravity, here on earth, is the same as gravity out in the solar system...

This led him to planetary orbits, and "celestial mechanics"...

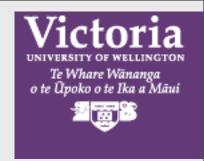




Newton's
gravity
is an
excellent
approximation
for
solar system
dynamics...

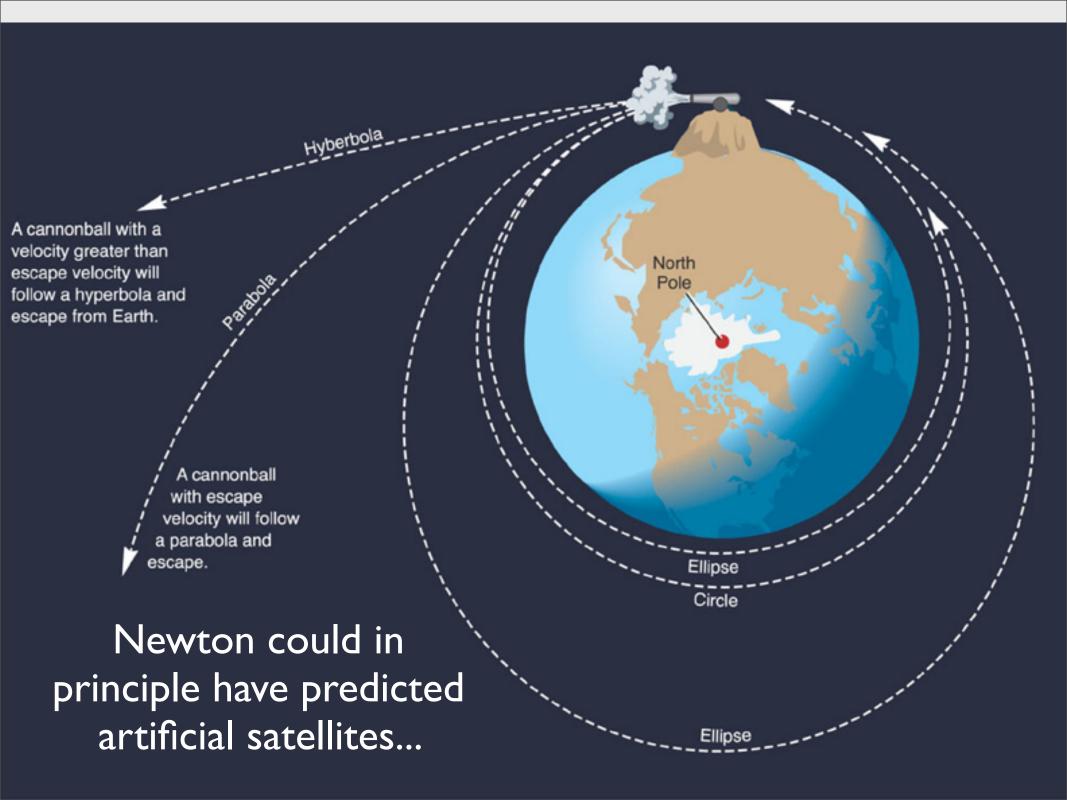
(Mercury, Pioneer anomaly...)





Solar system orbits, planets, asteroids, comets, etc, are circles, ellipses, parabolas, or hyperbolas...

(at least to a first approximation, until you look at the way planets, asteroids, comets, etc interact with each other...)





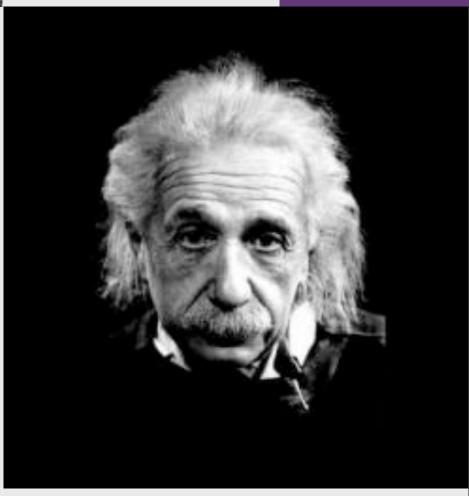
#### <u>Curved</u> <u>space-time:</u>



Enter Einstein...

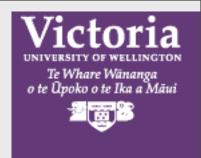
Euclid's geometry is not exactly "real world"...

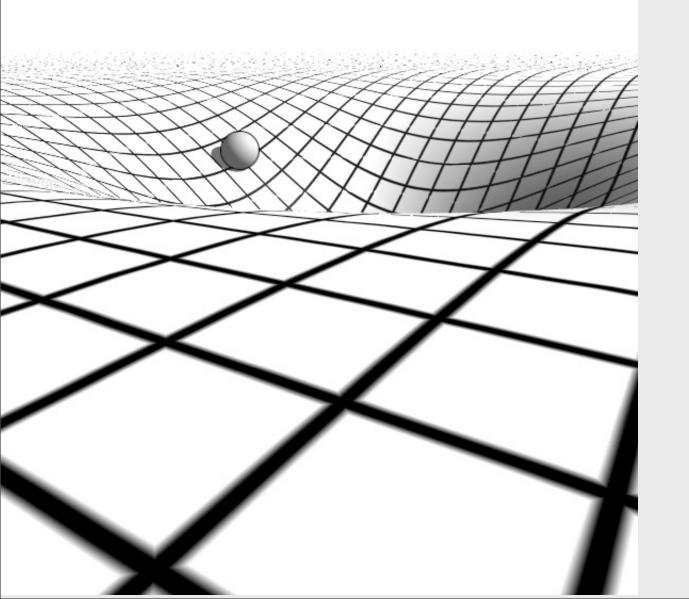
(Euclid's geometry is correct mathematics, but that does not make it "real world physics"...)



To more accurately describe physical reality you need Riemann's curved space+time...







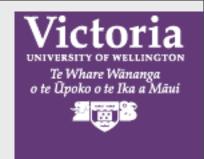
Space tells matter how to move...

Matter tells space how to curve...

$$G_{ab} = 8\pi G_N T_{ab}$$

Looks innocent, doesn't it?





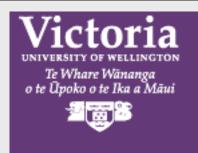
$$G_{ab} = 8\pi G_N T_{ab}$$

The Einstein equations actually a set of ten coupled nonlinear second-order partial differential equations...

Finding physically relevant exact solutions is often extremely difficult...



#### **Predictions:**



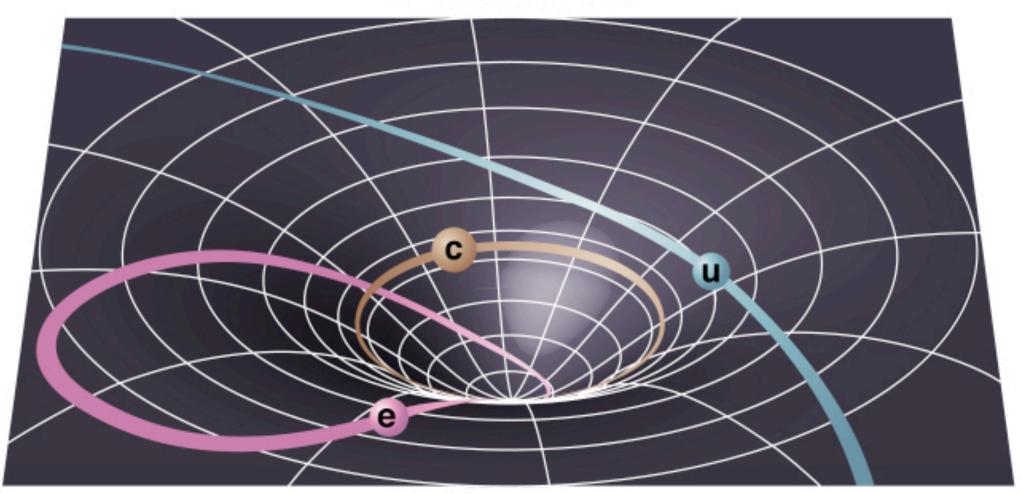
Approximately!

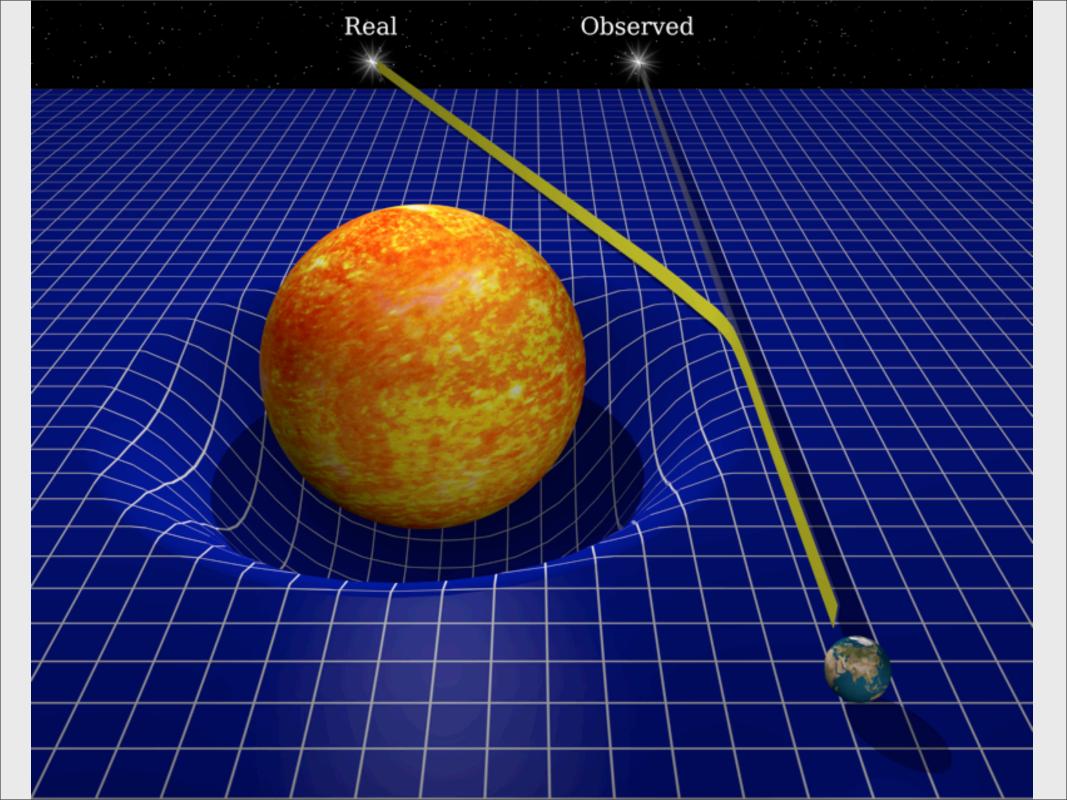
c circular orbit

e elliptical orbit

u unbound orbit

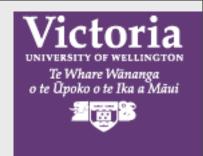
Approximately!

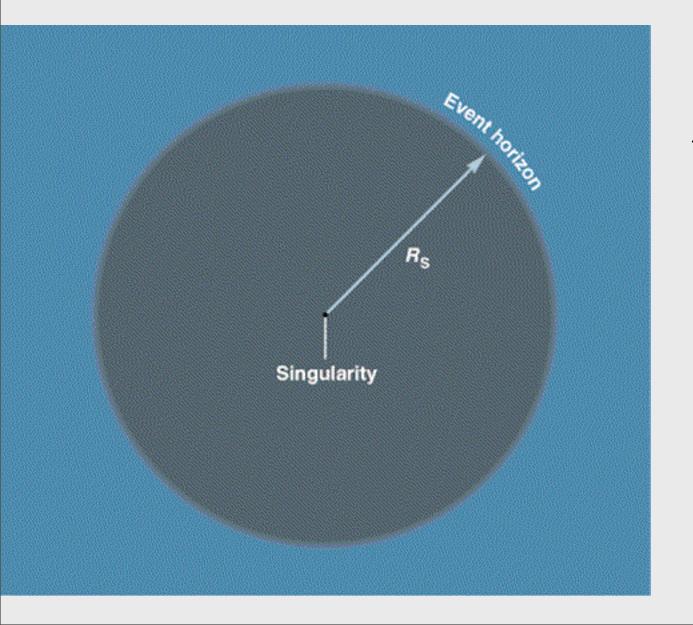




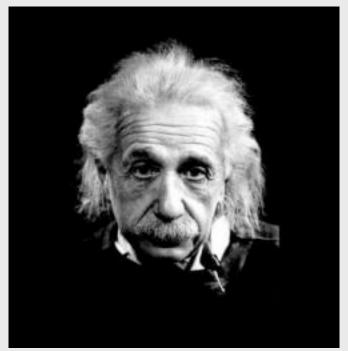


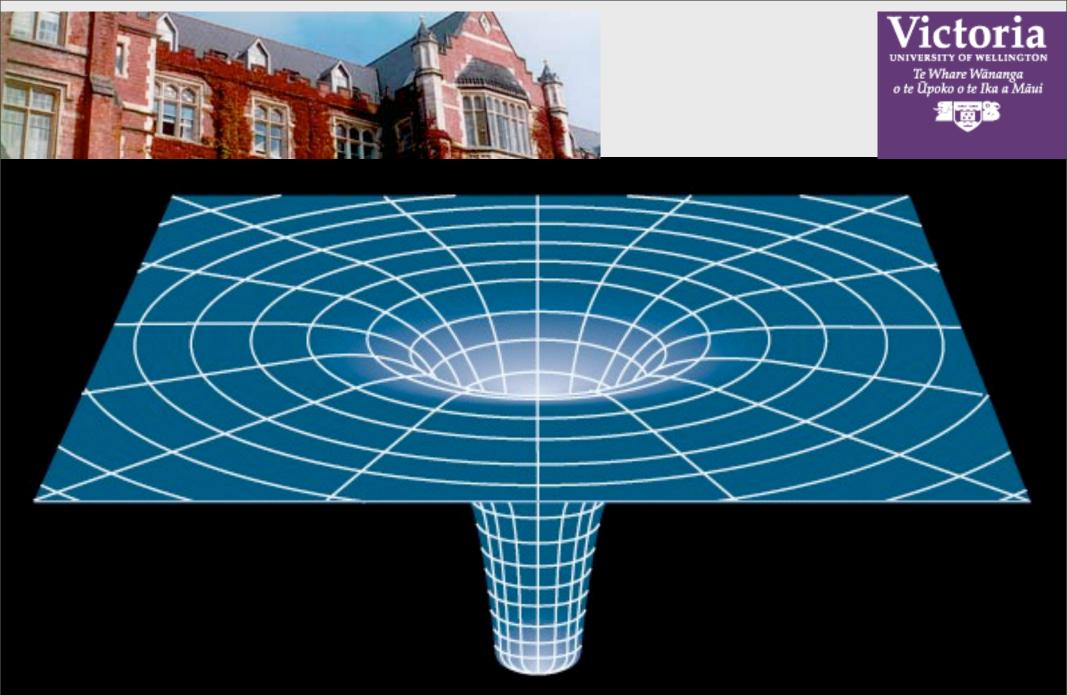
#### Black holes:





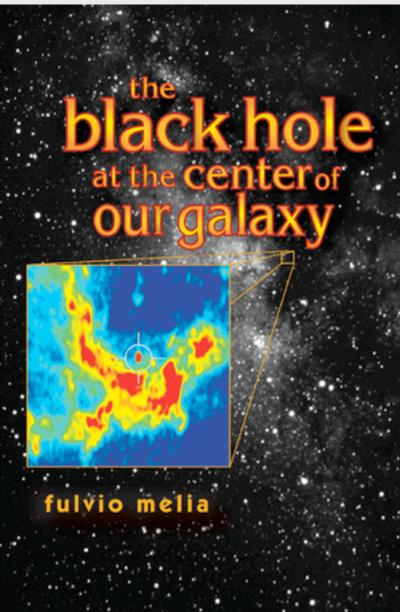
$$R = \frac{2GM}{c^2}$$



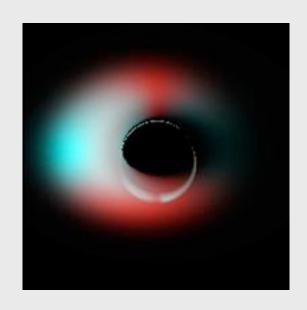


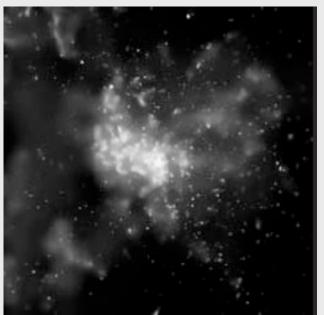
Classically, nothing can escape...

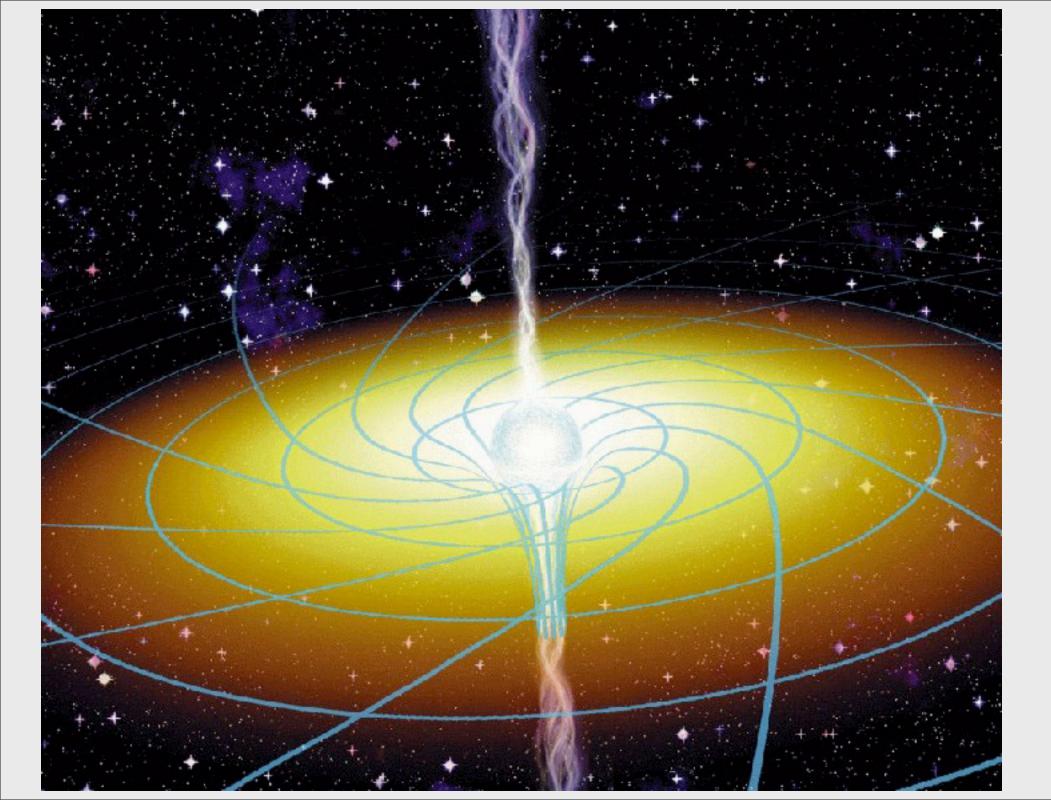




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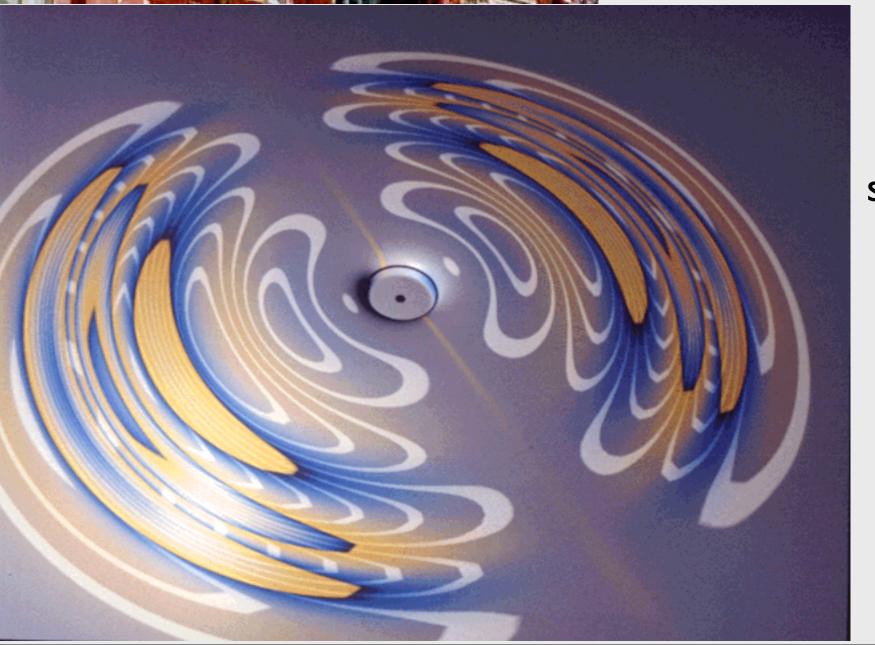






### Gravity waves





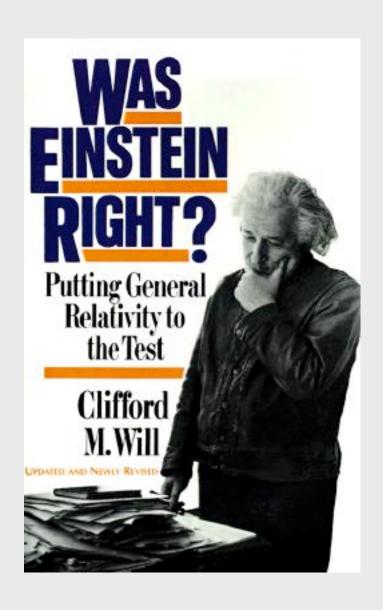
Ripples in space-time

Still collecting data...

LIGO







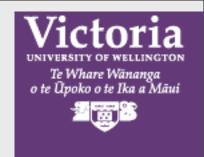
Short answer:

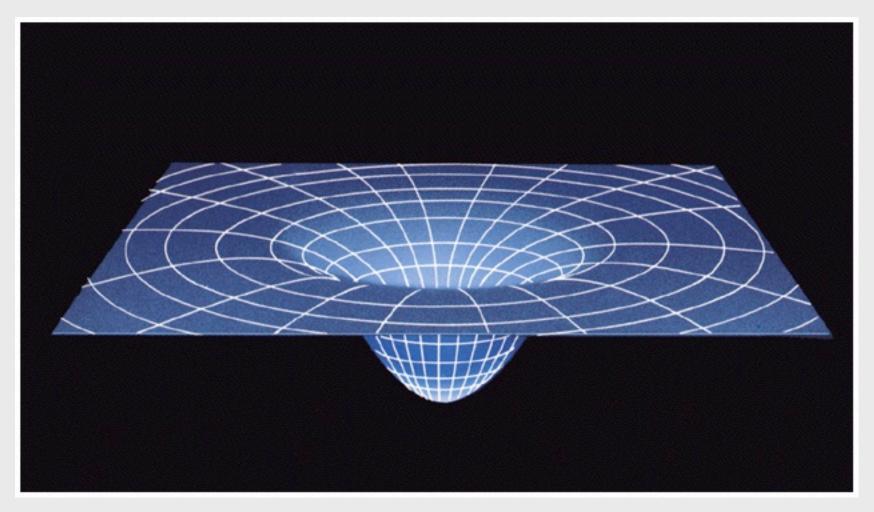
Yes...

Long answer:

Standard special and general relativity are completely compatible with present-day experiment...

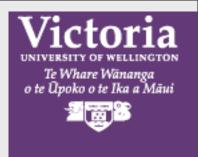






Spacetime curves --- in the manner Einstein predicted.





# Now add quantum physics...





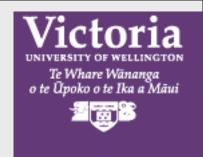
#### Why do we need quantum gravity?

There is this thing called the Planck scale.

First discussed by Max Planck in 1899.

Quantum physics was in its infancy.



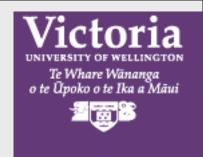


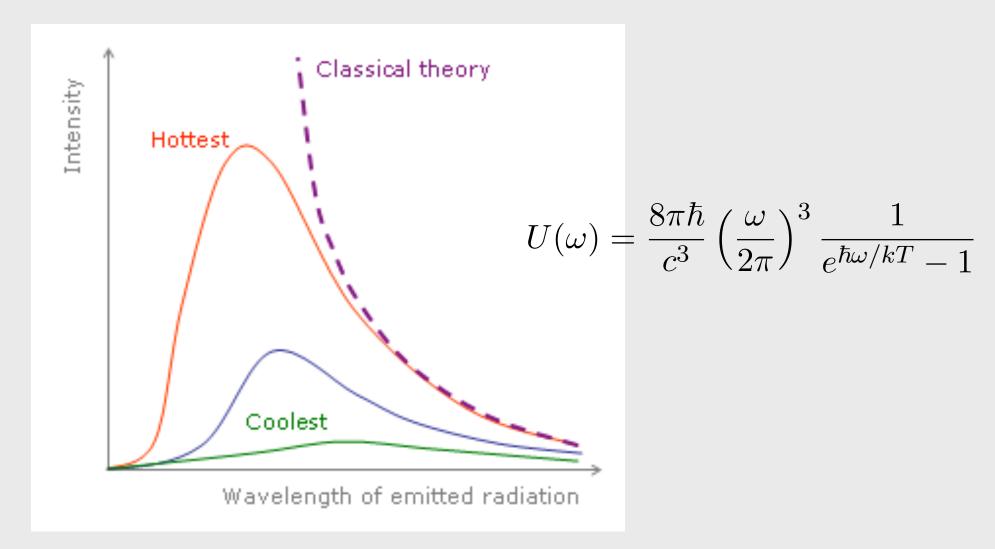
Planck constant — then only an empirical way of parameterizing the unexpected behaviour of black-body radiation.

Planck scale — seemed to be merely an accident of "algebraic numerology"

— you put  $\hbar$ , c, and G together in various ways and out popped masses, times, and distances.



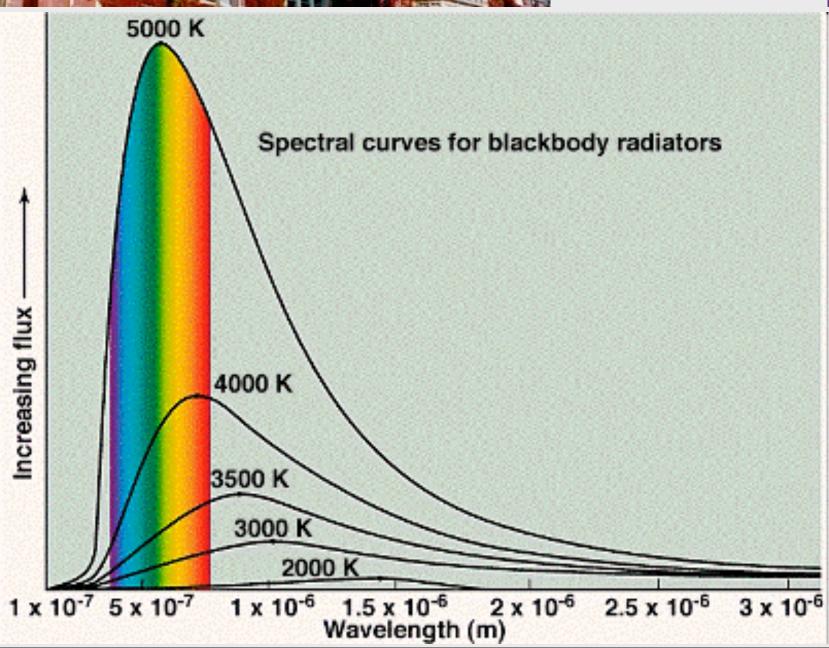




Black body spectrum







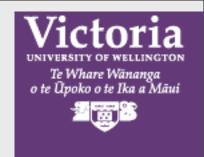




# Planck scale

Only after the development of quantum physics (Schrodinger, Heisenberg, 1925) was the significance of the Planck scale as the harbinger of quantum gravity appreciated.



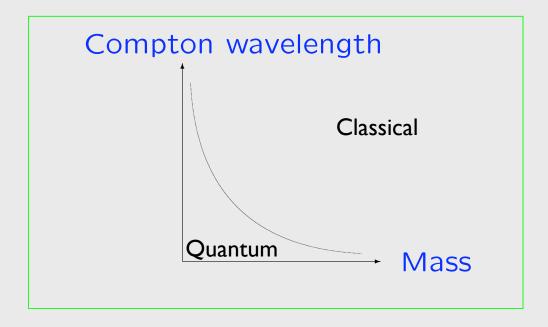


### The Planck scale: Compton wavelength

Quantum mechanics tells us that an elementary particle of mass M can be reasonably easily localized within a distance

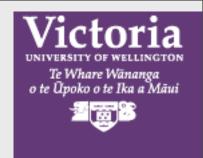
$$\lambda_{\mathsf{Compton}} = \frac{\hbar}{M \ c}$$

known as the Compton wavelength.



Compton wavelength as a function of mass.

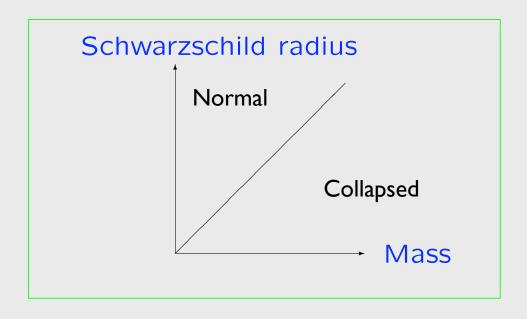




### The Planck scale: Schwarzschild radius

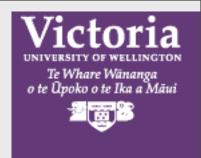
Classical gravity tells us that a particle of mass M will disappear down a black hole if the particle is smaller than its Schwarzschild radius

$$r_{\text{Schwarzschild}} = \frac{2 G M}{c^2}.$$



Schwarzschild radius as a function of mass.

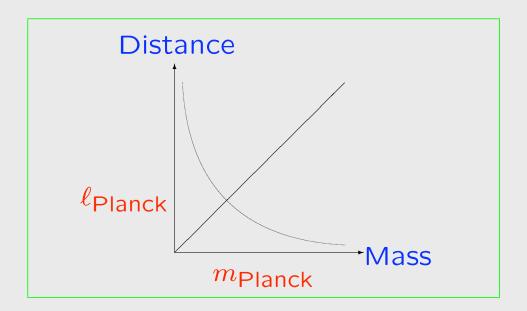




### The Planck scale: Crossover

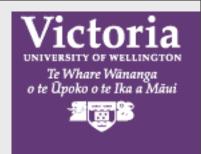
Plot the Compton wavelength as a function of mass, and the Schwarzschild radius as a function of mass.

The Planck mass is the place that the two graphs cross.



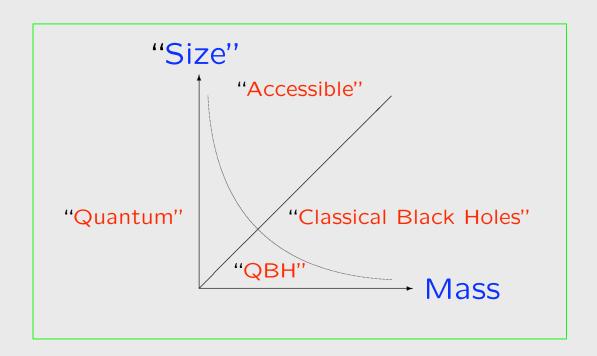
The Planck scale is the crossing point of Compton wavelength and Schwarzschild radius as a function of mass.



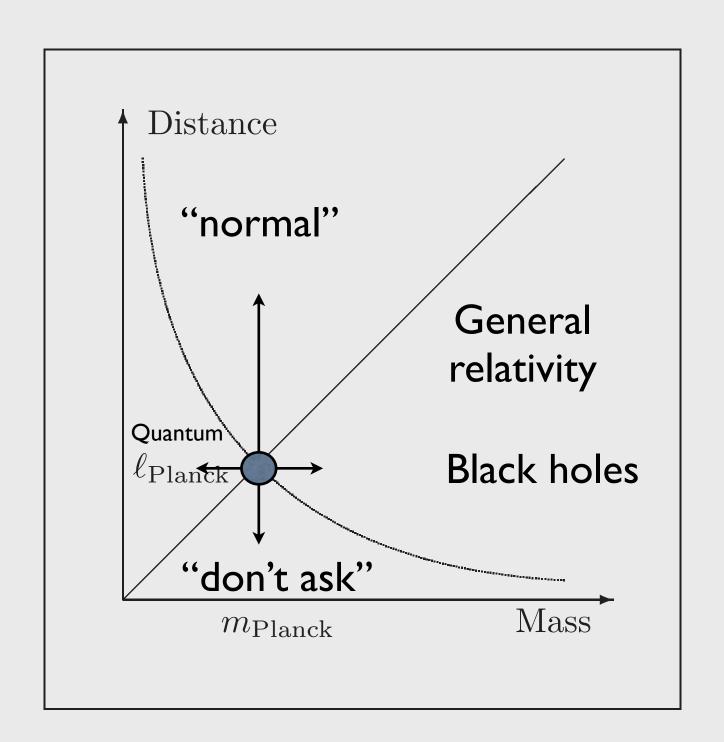


## Classical—Quantum transition:

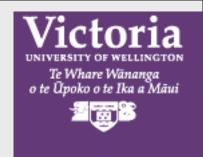
• The physically accessible region.



Accessible region for "effective radius" as a function of mass.







Eventually, we will have no choice...

Eventually, we will simply have to address the problem of "quantum gravity"...

What is the quantum theory that reduces to Einstein gravity in some appropriate limit?





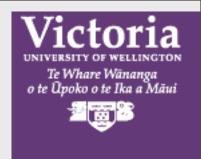
Just don't ask too many questions...

Because we are not so sure of the answers...

(at least, not yet...)

String theory, brane theory,
loop quantum gravity,
Lorentzian lattice quantum gravity,
Horava-Lifschitz gravity...





# "It is important to keep an open mind; just not so open that your brains fall out"

--- Albert Einstein