School of Mathematical and Computing Sciences
Te Kura Pangarau, Rorohiko

Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

Analogue spacetimes: Toy models for "quantum gravity".

Matt Visser

TU Muenchen

Tuesday 14 August 2007

Abstract:

Why are "analogue spacetimes" interesting?

The answer is actually rather simple:

Analogue spacetimes provide one with physically well-defined and physically well-understood concrete models of many of the phenomena that seem to be part of the yet incomplete theory of "quantum gravity", or more accessibly, "quantum gravity phenomenology".

Abstract:

For example "analogue spacetimes" provide concrete models of "emergence" (the effective low-energy theory can be radically different from the high-energy microphysics).

Provide controlled models of "Lorentz symmetry breaking", extensions of the usual notions of Lorentzian geometry: "rainbow spacetimes", pseudo-Finsler geometries, and more...

I will provide an overview of the key items of "unusual physics" that arise in analogue spacetimes, and argue that they provide us with hints of what we should be looking for in any putative theory of "quantum gravity".

Silke Weinfurtner: Victoria University of Wellington
New Zealand

Stefano Liberati: SISSA / ISAS, Trieste, Italy

Carlos Barcelo: Instituto Astrofisica de Andalusia
Granada, Spain

Angela White: ANU,   Canberra

Piyush Jain: VUW,   NZ

Crispin Gardiner: Otago University,   NZ

The word "emergence" is being tossed around
an awful lot lately.....

But what does it really mean?

--- "More is different"?

--- The sum is greater than its parts?

--- Universality?

--- Mean field?

Short distance physics is often radically different
from long distance physics...

Prime example: Fluid dynamics

Long distance physics: Euler equation   (generic)

Continuity equation   (generic)

Equation of state   (specific)

Short distance physics: Quantum molecular dynamics

Note: You cannot hope to derive quantum molecular dynamics by quantizing fluid dynamics...

Could Einstein gravity be "emergent"?

1) Can we get an "analogue spacetime"?   (generic)

2) Can we get Einstein's equations?   (specific)

IF Einstein gravity is "emergent",
THEN it makes absolutely no sense
to "quantize gravity" as a "fundamental" theory...

The best one could then hope for is some uber-theory that
approximately reduces to Einstein gravity in some limit.

The uber-theory would not necessarily be quantum...

It must have as approximate limits: ['t Hooft]

--- Classical Einstein gravity...

--- Quantum field theory (Minkowski)...

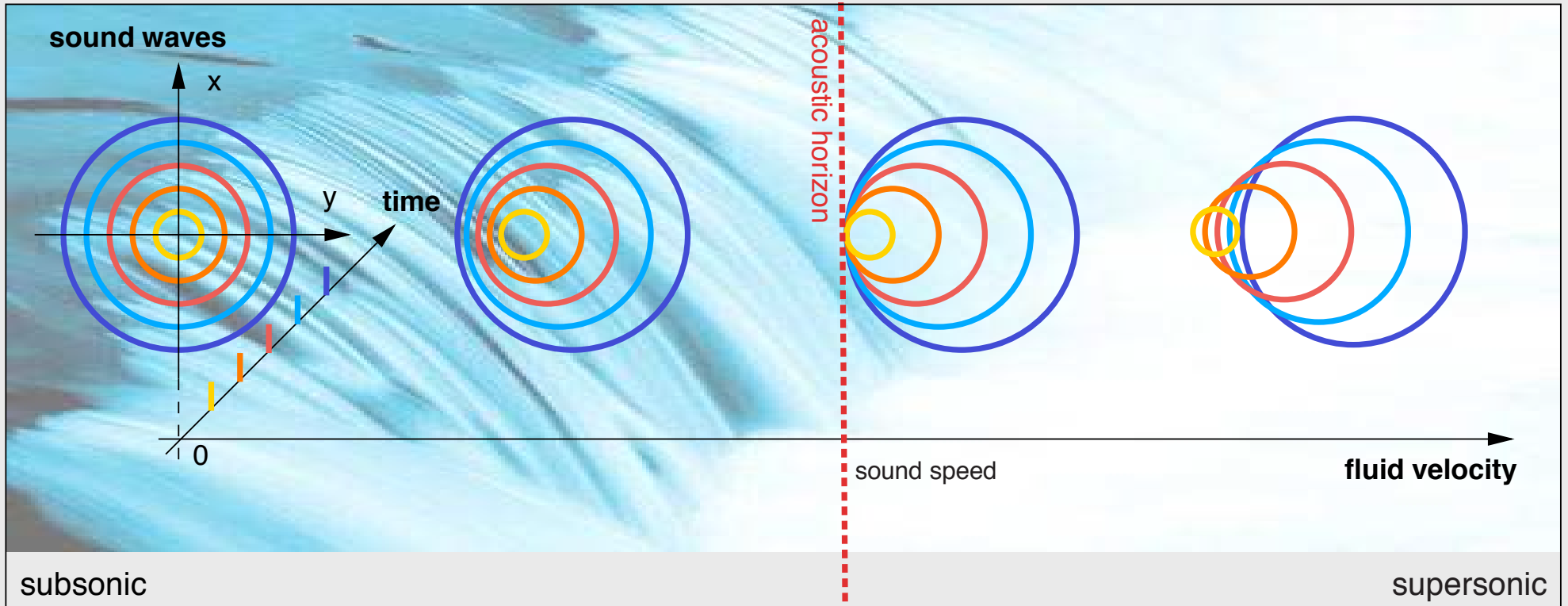--- Curved space QFT...

--- Semiclassical quantum gravity...

Analogue spacetimes are (among other things)
baby steps in this direction...

The simplest "analogue spacetimes" are the "acoustic spacetimes"...

Consider sound waves in a moving fluid...   [Unruh 81]

Theorem: Consider an irrotational, inviscid, barotropic perfect fluid, governed by the Euler equation, continuity equation, and an equation of state.

The dynamics of the linearized perturbations (sound, phonons) is governed by a D'Alembertian equation

$$\Delta_g \Phi = \frac{1}{\sqrt{g}} \partial_a \left(\sqrt{g} \, g^{ab} \partial_b \, \Phi \right) = 0$$

involving an "acoustic metric".

[Algebraic function of the background fields.]

Theorem:

<div align="right">(3+1 dimensions)</div>

$$g^{\mu\nu}(t,\vec{x}) \equiv \frac{1}{\rho_0 c} \left[\begin{array}{c:c} -1 & -v_0^j \\ \hdashline -v_0^i & (c^2\,\delta^{ij} - v_0^i\,v_0^j) \end{array} \right].$$

$$g_{\mu\nu}(t,\vec{x}) \equiv \frac{\rho_0}{c} \left[\begin{array}{c:c} -(c^2 - v_0^2) & -v_0^j \\ \hdashline -v_0^i & \delta_{ij} \end{array} \right].$$

$$\mathrm{d}s^2 \equiv g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \frac{\rho_0}{c}\left[-c^2\,\mathrm{d}t^2 + (\mathrm{d}x^i - v_0^i\,\mathrm{d}t)\,\delta_{ij}\,(\mathrm{d}x^j - v_0^j\,\mathrm{d}t)\right].$$

There is by now a quite sizable literature on acoustic, and other more general analogue spacetimes

Unruh: Experimental black hole evaporation,
Phys Rev Lett 46 (1981) 1351-1353.

Barcelo, Liberati, Visser: Analogue gravity,
Living Reviews in Relativity, 8:12, 2005.

Main message: Finding an effective low-energy metric is not all that difficult....

Examples of exotic physics:

Controlled signature change   [White, Weinfurtner]

Bose-nova   [Hu, Calzetta]

c^2 propto   (scattering length)

Can be controlled by using a Feschbach resonance.

There is no general widely accepted precise mathematical definition of what is meant by a "rainbow geometry"...

The physicist's definition is rather imprecise:
"energy dependent metric"?
"momentum dependent metric"?
"4-momentum dependent metric"?

Q: 4-momentum of what? The observer?
The object being observed?

To capture the essence of "energy dependence" need a metric that depends also on the magnitude of the tangent vector....

Consider a fluid at rest, in very many cases the dispersion relation can be written in the form:

$$\omega^2 = F(k)$$

for some possibly nonlinear function F(k)...
(2nd-order in time; arbitrary order in space...)

[Unruh, Jacobson]

Phase velocity:

$$c_k^2 = \frac{\omega^2}{k^2} = \frac{F(k)}{k^2}$$

Dispersion relation:

$$\omega^2 = c_k^2 \, k^2$$

Fluid in motion: Doppler shift the frequency...

$$\omega \longrightarrow \omega - \vec{v} \cdot \vec{k}$$

$$\left(\omega - \vec{v} \cdot \vec{k}\right)^2 - c_k^2 \, k^2 = 0$$

Rewrite as: $$g_k^{ab} \, k_a \, k_b = 0.$$

Pick off components:

$$g_k^{ab} \propto \left[\begin{array}{c|c} -1 & -v^j \\ \hline -v^i & c_k^2 \, \delta^{ij} - v^i v^j \end{array} \right].$$

$$g_{ab}^k \propto \left[\begin{array}{c|c} -(c_k^2 - v^2) & -v^j \\ \hline -v^i & \delta^{ij} \end{array} \right].$$

Momentum dependent metric depending on phase velocity.

Dispersion relation approach is physically transparent...

Only weakness: Conformal factor left unspecified...

(This is a standard side-effect of the geometrical
quasi-particle approximation,
cf geometrical acoustics,
cf geometrical optics.)   [PDE is better]
[Weinfurtner]

The momentum in question is now the momentum
of an individual "mode" of the field ---
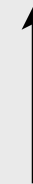hence phase velocity + dispersion relation.

Similar (but distinct) steps can be taken to develop a rainbow metric based on group velocity.

Consider a wave packet centered on momentum k.

That packet will propagate with the group velocity.

$$(\mathrm{d}\vec{x} - \vec{v}\,\mathrm{d}t)^2 = c_k^2\,\mathrm{d}t^2$$

Group velocity.

Rewrite as:
$$\mathrm{d}s^2 = 0 = g_{ab}\, dx^a\, \mathrm{d}x^b$$

Pick off components:

$$g_k^{ab} \propto \left[\begin{array}{c|c} -1 & -v^j \\ \hline -v^i & c_k^2\, \delta^{ij} - v^i v^j \end{array}\right].$$

$$g_{ab}^k \propto \left[\begin{array}{c|c} -(c_k^2 - v^2) & -v^j \\ \hline -v^i & \delta^{ij} \end{array}\right].$$

Momentum dependent metric depending on group velocity.

There are at least two distinct very different notions of "Rainbow metric" in an analogue setting.

They answer different questions:

* What is the dispersion relation of a pure mode?

* How do wave packets propagate?

If you are lucky there is a "hydrodynamic" limit:

$$\lim_{k \to 0} c^2_{\text{phase}}(k) = c^2_{\text{hydrodynamic}} = \lim_{k \to 0} c^2_{\text{group}}(k)$$

$$\neq 0!$$

In general: Rainbow ==> multi-metric

$$g_k^{ab} \propto \left[\begin{array}{c|c} -1 & -v^j \\ \hline -v^i & c_k^2\,\delta^{ij} - v^i v^j \end{array} \right].$$

$$g_{ab}^k \propto \left[\begin{array}{c|c} -(c_k^2 - v^2) & -v^j \\ \hline -v^i & \delta^{ij} \end{array} \right].$$

With: $\quad c_k \to \begin{cases} c_{\text{phase}} \\ c_{\text{group}} \\ c_{\text{hydrodynamic}} \end{cases}$.

Signal speed?

c ==> infinity?

Bogoliubov dispersion relation (eg, BECs):

$$\omega^2 = c_0^2\, k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$

$$c^2 = c_0^2 + \left(\frac{\hbar}{2m}\right)^2 k^2 \qquad \text{(supersonic)}$$

Controlled breaking of Lorentz invariance...

See "quantum gravity phenomenology"...   [Liberati...]

See "cosmological particle production" [Weinfurtner]

Surface waves in finite depth of liquid: [Lamb]

$$\omega^2 = g\,k\,\tanh(k\,d) = c_0^2\,k^2\,\frac{\tanh(k\,d)}{k\,d} \qquad c_0^2 = g\,d.$$

$$c^2 = c_0^2\,k^2\,\frac{\tanh(k\,d)}{k\,d} \qquad \text{(subsonic)}$$

$$\omega^2 = c_0^2\,k^2\,\left\{ 1 - \frac{(k\,d)^2}{3} + \frac{2(k\,d)^2}{15} + \dots \right\}$$

So analogue models provide concrete examples for both supersonic an subsonic dispersion, and more...

Surface waves in infinite depth of liquid:

$$\omega = \sqrt{g\,k}; \qquad c_{\mathrm{phase}} = \sqrt{g/k}.$$

$$c_{\mathrm{group}} = \frac{\partial \omega}{\partial k} = \frac{\sqrt{g/k}}{2} = \frac{c_{\mathrm{phase}}}{2}.$$

No hydrodynamic limit...

No well-defined low-momentum spacetime...

[You could argue that this is an unphysical limit...]

Surface waves in finite depth of liquid + surface tension:

$$\omega^2 = c_0^2 \, k^2 \left\{ 1 + \frac{\sigma}{\rho \, c_0^2 \, d}(kd)^2 \right\} \frac{\tanh(kd)}{kd}.$$

$$c^2 = c_0^2 \left\{ 1 + \frac{\sigma}{\rho \, c_0^2 \, d}(kd)^2 \right\} \frac{\tanh(kd)}{kd}.$$

$$c_0^2 = g \, d.$$

Asymptotically supersonic, though it can be adjusted to have a subsonic dip.

Water: $\quad \epsilon = \dfrac{\sigma}{\rho c_0^2 d} = \dfrac{\sigma}{\rho g d^2} = \dfrac{(0.27 \ \mathrm{cm})^2}{d^2}.$

$$c^2 = c_0^2 \left\{ 1 + \epsilon \, (kd)^2 \right\} \, \frac{\tanh(kd)}{kd}.$$

$$c^2 = c_0^2 \left\{ 1 + \frac{3\epsilon - 1}{3} \, (kd)^2 - \frac{5\epsilon - 2}{15} \, (kd)^4 + \mathcal{O}[(kd)^6] \right\}.$$

Can tune away the lowest order Lorentz violation...

(Water at 0.47 cm depth)

These are just some examples of the types of dispersion relation you can arrange...

Can also arrange for particle masses:

$$\omega^2 = \omega_0^2 + c_0^2\, k^2 + \frac{k^4}{K^2} + \mathcal{O}[(k)^6].$$

[2 interacting BECs: Weinfurtner *et al...*]

Basic message: Lots of physically well behaved and well controlled toy models for many different types of "beyond the standard model" physics...

Many interesting extensions and modifications of the general relativity notion of spacetime have concrete and well controlled models within the "analogue spacetime" framework.

This tells us which rocks to start looking under...

"It is important to keep an open mind; just not so open that your brains fall out"

--- Albert Einstein