Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui

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Can we hope to justify the Einstein equations in effective/ analogue/ emergent spacetimes?

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Effective models of quantum gravity
Perimeter Institute
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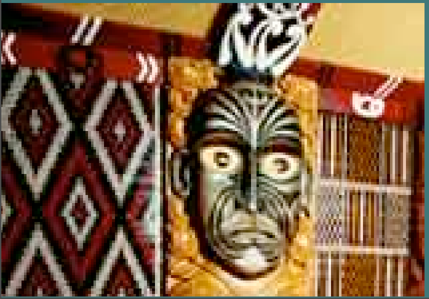

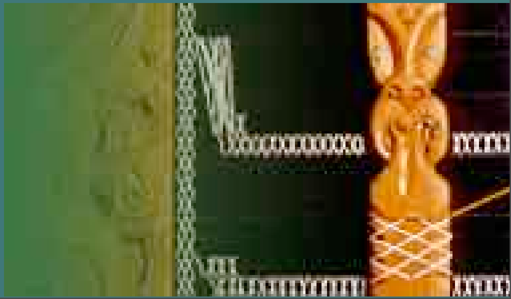

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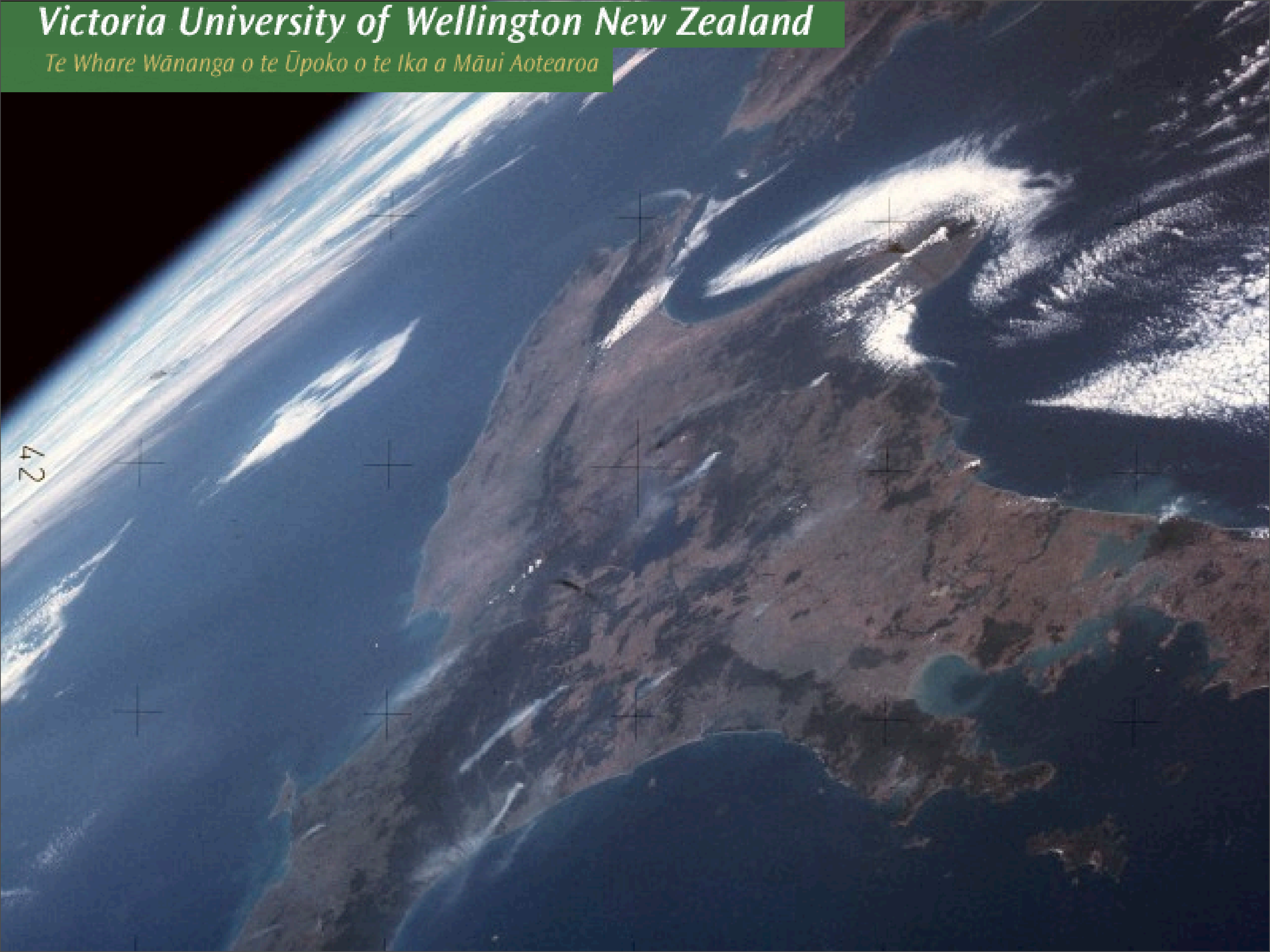

Abstract:

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Analogue/ emergent spacetimes currently are useful for describing kinematic aspects of quantum gravity, that is: How do particles and fields react to the presence of the analogue/ emergent spacetime? But obtaining suitable Einstein-like dynamics for the analogue/ emergent spacetime is certainly much more difficult, and may in most (hopefully not all) analogue models prove to be impossible. Without providing any definitive solution to this problem, I will try to explore the possibilities and summarize the current situation.

Health and safety advisory:

For this talk I will be in “wild speculation mode”.

I’ll be talking about hopes and fears,
guesses and possibilities...
Probabilities?

Treat all formulae as correct up to signs,
and a few coefficients.

For all effective/ emergent/ analogue attempts at attacking quantum gravity there are two distinct and fundamentally different questions to be asked:

I) What is the arena in which the physics takes place?

(This will define your effective/ emergent/ analogue spacetime, and permit you to formulate kinematical questions.)

II) What is the dynamics controlling this area?

(Ultimately, can you extract anything like some approximation to the Einstein equations?)

I will assume you somehow have found/ built/ created your arena --- spacetime --- and focus on questions of dynamics...

The only really generic way of developing Einstein-like spacetime dynamics *ab initio* seems to be through some variant of Sakharov's "induced gravity".

(Certainly for "analogue spacetimes" there seems to be little chance of any other route panning out.)

Fundamental observation: One-loop physics induces a Einstein-Hilbert term.

[Utterly standard]

More precisely, one-loop physics generates a slew of terms proportional to the various Seeley-DeWitt coefficients.

Before renormalization, for each individual particle species:

$$a_0\ \kappa^4 + (a_1 - m^2\ a_0)\ \kappa^2 + \left(a_2 - m^2 a_1 + \frac{1}{2} m^4 a_0\right) \ln\left(\frac{\kappa}{m}\right)$$

That is:

$$a_0\ \left\{\kappa^4 - m^2\kappa^2 + \frac{1}{2}m^4\ \ln\left(\frac{\kappa}{m}\right)\right\} + a_1 \left\{\kappa^2 - m^2\ \ln\left(\frac{\kappa}{m}\right)\right\} + a_2\ \ln\left(\frac{\kappa}{m}\right)$$

Now sum over all particle species...

[Utterly standard]

Let us generically write: $$a_{J,i} = \sum_{\sigma} k_{J,\sigma,i}\, \mathcal{A}_{J,\sigma}$$

This is the J'th Seeley-DeWitt coefficent,
for the i'th species.

The "k" are dimensionless constants,
the "A" are a suitable basis.

$$\mathcal{A}_{0,\sigma} = 1; \qquad \mathcal{A}_{1,\sigma} = \mathcal{R};$$

$$\mathcal{A}_{2,\sigma} = \{C_{abcd}\, C^{abcd}, \quad \underset{\mathsf{X}}{R_{ab}\, R^{ab}}, \quad R^2, \quad \underset{\mathsf{X}}{\Box R}, \quad F_{ab}\, F^{ab}\}$$

[Utterly standard]

Then to one loop: $(k_{0,i} \equiv 1)$

$$\Delta(\rho_\Lambda) = \sum_i (-)^{2s_i} \; g_i \; \left\{ \kappa^4 - m_i^2 \kappa^2 + \frac{1}{2} m_i^4 \; \ln\left(\frac{\kappa}{m_i}\right) \right\}$$

$$\Delta(M^2_{\text{Planck}}) = \sum_i (-)^{2s_i} \; g_i \; k_{1,i} \left\{ \kappa^2 - m_i^2 \; \ln\left(\frac{\kappa}{m_i}\right) \right\}$$

$$\Delta(\lambda_\sigma) = \sum_i (-)^{2s_i} \; g_i \; k_{2,i,\sigma} \; \ln\left(\frac{\kappa}{m_i}\right)$$

Sakharov's proposal (as far as anyone can tell), was intended to be a little more radical...

[Sakharov]

Assume there is *no* zero-loop term...

Then to one loop:

$$\rho_\Lambda = \sum_i (-)^{2s_i} \; g_i \; \left\{ \kappa^4 - m_i^2 \kappa^2 + \frac{1}{2} m_i^4 \; \ln\left(\frac{\kappa}{m_i}\right) \right\}$$

$$M^2_{\text{Planck}} = \sum_i (-)^{2s_i} \; g_i \; k_{1,i} \left\{ \kappa^2 - m_i^2 \; \ln\left(\frac{\kappa}{m_i}\right) \right\}$$

$$\lambda_\sigma = \sum_i (-)^{2s_i} \; g_i \; k_{2,i,\sigma} \; \ln\left(\frac{\kappa}{m_i}\right)$$

[cutoff dependent]

[Radical Sakharov]

Assume the cutoff dependence actually cancels, and so everything is finite...

This constrains the particle physics spectrum.

Cosmological constant:

$$\sum_i (-)^{2s_i} \; g_i = 0 \qquad\qquad \rho_\Lambda = -\frac{1}{2} \sum_i (-)^{2s_i} \; g_i \; m_i^4 \; \ln \left(\frac{m_i}{\mu} \right)$$

$$\sum_i (-)^{2s_i} \; g_i \; m_i^2 = 0$$

[3 constraints, 1 output]

$$\sum_i (-)^{2s_i} \; g_i \; m_i^4 = 0$$

This result known to Pauli...

[SUSY is sufficient]

[Radical
Sakharov]

Newton constant:

$$\sum_i (-)^{2s_i} \; g_i \; k_{1,i} = 0$$

[2 constraints, 1 output]

$$\sum_i (-)^{2s_i} \; g_i \; m_i^2 \; k_{1,i} = 0$$

[SUSY *not* sufficient]

$$M^2_{\rm Planck} = -\sum_i (-)^{2s_i} \; g_i \; m_i^2 \; k_{1,i} \; \ln\left(\frac{m_i}{\mu}\right)$$

If you could really trust this beyond 1 loop,
this would be truly stunning...

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[Radical
Sakharov]

Curvature-squared couplings (including gauge fields):

(While you are at it, why stop with just gravity?)

$$\sum_i (-)^{2s_i}\; g_i\; k_{2,i,\sigma} = 0$$ [1 constraint per coupling]

$$\lambda_\sigma = -\sum_i (-)^{2s_i}\; g_i\; k_{2,i,\sigma}\; \ln\left(\frac{m_i}{\mu}\right)$$

[SUSY *not* sufficient]

[Radical Sakharov]

I think you can see why people keep returning to Saharov's ideas...

Details, problems, are tricky...

Potential payoff huge...

Certainly in "radical Sakharov", the Newton (and other) physical constants become in principle *calculable* features of the particle spectrum...

Everyone loves a zeta function...

Purely for promotional and propaganda purposes, define a zeta function:

$$\zeta_{J,\sigma} = \sum_i (-)^{2s_i} \, g_i \, k_{J,i,\sigma} \, \left(\frac{m_i}{\mu}\right)^{-s}$$

Finiteness constraints:

$$\zeta_0(0) = 0$$

$$\zeta_1(0) = 0$$

$$\zeta_0(-2) = 0$$

$$\zeta_{2,\sigma}(0) = 0$$

$$\zeta_1(-2) = 0$$

$$\zeta_0(-4) = 0$$

Everyone loves a zeta function...

Physical constants:

$$\rho_\Lambda = \frac{1}{2}\,\mu^4\,\zeta_0'(-4)$$

$$M_{\mathrm{Planck}}^2 = \mu^2\,\zeta_1'(-2)$$

$$\lambda_\sigma = \zeta_{2,\sigma}'(0)$$

[cf: zeta functions and Casimir energy]

This is all so very pretty that it’s almost a pity to start pointing out the problems...

(One-loop result, free field theory, and by the way, what about zero-loop physics?)

Note the positives:

Once you have a spacetime arena to play in,
almost anything will auto-magically
generate an Einstein-Hilbert term
in the effective action...

Note the negatives:

If you want to *calculate* the Planck scale *ab initio*,
then you have to make extremely strong
assumptions about *both* zero-loop
and multi-loop physics,
and about the spectrum of elementary particles.

This is extremely "iffy" in an "analogue model" setting.

The "zero-loop physics" is then (typically)
some variant of fluid mechanics, or
(as Sakharov himself was suggesting),
some sort of "crystal spacetime lattice".

The point is that in this context the fundamental
zero-loop physics is typically *not*
"Lorentz invariant", though fluctuations typically
are effectively "Lorentz invariant".

Phenomenologically, one needs to suppress
zero-loop physics...

Simple you say, take the zero-loop Lagrangian, multiply by some small parameter, and tune to zero...

$$\mathcal{L}_0 \to \epsilon\, \mathcal{L}_0; \qquad \epsilon \to 0$$

Unfortunately: $$\epsilon\, \mathcal{L}_0 \;=\; \frac{\mathcal{L}_0}{1/\epsilon}$$

Where "1/epsilon" is now the "loop-counting parameter", which is becoming large...

So suppressing zero-loop physics tends to enhance infinite-loop physics...

Overall: Not only is there no particularly good reason to stop calculating at one loop, but attempts at suppressing zero-loop physics will generically “seesaw” to amplify higher loop physics, where current formulations of the “heat kernel” (Seeley-DeWitt) method fail to give useful information.

Alternatives: Try to make zero-loop physics a topological field theory? But then the fluctuations are also topological?

Alternatives:

In flat spacetime we now have a relatively wide class of "finite to all loops" QFTs.

SUSY being neither necessary nor sufficient for all loop finiteness.

SUSY was/is historically useful in *finding* these all loop finite QFTs.

All known finite QFTs exhibit unbroken or "softly broken" SUSY.

Alternatives:

[Speculation]

These finite QFTs auto-magically satisfy the cosmological constant finiteness constraints, since these are already flat-space results [Pauli].

Maybe these finite QFTs can be extended to non-dynamical gravity...

(Gravity treated as an external field...)

This would make physical gravity "partially dynamical", classically dynamical only after integrating out the "other" [ie, quantum] fields...

[most relativists would choke...]

Alternatives:

[Speculation]

There is increasing evidence that N=8 SUGRA, like N=4 SUSY Yang-Mills, might actually be finite...

(N=8 SUGRA at least 3-loop finite...)

(Something more than SUSY seems to be going on...)

If N=8 SUGRA is indeed finite, then *maybe* it can be softly broken without destroying finiteness.

A softly broken but finite N=8 SUGRA should then auto-magically satisfy the "radical Sakharov" finiteness constraints...

[one-loop finiteness would actually be good enough...]

Alternatives:

[Speculation]

Then maybe loop corrections to Newton's constant become calculable?

Maybe:

$$\frac{\hbar c}{G} = \frac{\hbar c}{G_0} - \hbar \sum_i (-)^{2s_i} \; g_i \; m_i^2 \; k_{1,i} \; \ln\left(\frac{m_i}{\mu}\right) + \mathcal{O}(\hbar^2)$$

(are we worried yet?)

Or maybe you should just think of these as consistency conditions?

So, can we hope to justify the Einstein equations in effective/ analogue/ emergent spacetimes?

Maybe:

Getting an Einstein-Hilbert term is not too difficult.

Getting the rest of the physics right, or at least not hopelessly wrong, is *much* trickier.

"It is important to keep an open mind; just not so open that your brains fall out"

--- Albert Einstein