Charge-nonconserving decays in ordinary matter

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A geophysical technique for placing experimental limits on charge-nonconserving decays is discussed, with the results \( \omega(n \rightarrow p + \text{neutrons}) < 2 \times 10^{-22} \text{ yr}^{-1} \), \( \omega(n \rightarrow p + e^+ + \text{neutrons}) < 10^{-22} \text{ yr}^{-1} \). These limits hold for bound neutrons, averaged over the chemical composition of the Earth.

In standard grand unification models the only gauge symmetries that remain unbroken are the U(1) of QED and the SU(3) of quantum chromodynamics. However, it is not inconceivable that in the real world the U(1) of QED is spontaneously broken at some level; presumably this would lead to a massive photon and to charge-nonconserving interactions. It therefore behooves us to look at the experimental evidence regarding charge conservation. Surprisingly, the limits on charge-nonconserving decays are less rigorous than the limits on baryon-nonconserving decays.

GEOPHYSICS

To study charge nonconservation we use a modification of a technique due to Pomansky.\(^1\) Consider the planet Earth (rock plus oceans, excluding the atmosphere); this is an object consisting mainly of electrons (\(e\)), protons (\(p\)), and neutrons (\(n\)). Observe that due to kinematics, the only possible decay of the electron is via the charge-nonconserving channel: \(e \rightarrow \text{neutrons (photons, neutrinos)}, \) with rate \(\omega_e\). Let us define

\[
\omega_p = \sum_{n=0, \pm 1, \pm 2, \ldots} \omega_p(\Delta Q = n),
\]

\[
(\Delta Q_p) = \text{average value of the difference between the final and the initial charge as a result of proton decay}
\]

\[
= \sum_{n=0, \pm 1, \pm 2, \ldots} n \omega_p(\Delta Q = n),
\]

with similar expressions for the neutron:

\[
\omega_n = \sum_{n=0, \pm 1, \pm 2, \ldots} \omega_n(\Delta Q = n),
\]

\[
(\Delta Q_n) = \sum_{n=0, \pm 1, \pm 2, \ldots} \omega_n(\Delta Q = n),
\]

Using these definitions,

\[
\frac{dQ}{dt} = e(N_e \omega_e + N_p (\Delta Q_p) \omega_p + N_n (\Delta Q_n) \omega_n) + I,
\]

where \(N_e = N_p = N_n \equiv \frac{1}{2}(M_{\text{Earth}}/M_n), \) and \(I\) is the net current flowing into the Earth. Rearranging the above yields

\[
\frac{dQ}{dt} = I + \frac{1}{2}N_e \omega_e + \frac{1}{2}N_p (\Delta Q_p) \omega_p + \frac{1}{2}N_n (\Delta Q_n) \omega_n,
\]

Using Gauss's law: \(Q = \epsilon \int \mathbf{E} \cdot d\mathbf{S}, \) we may use observations of the electric field at the surface of the Earth to deduce \(Q\). Measurements of the fine weather field have been made since 1752,\(^2\) and lead to an average value of 100 V/m, corresponding to \(Q = -5 \times 10^5 \text{ C}. \) Fluctuations in \(E\) are of order 20%, so we may set

\[
\frac{dQ}{dt} < \frac{10^5 \text{ C}}{200 \text{ yr}} \approx 2 \times 10^{-4} \text{ A}.
\]

To estimate the current flowing into the Earth we must consider the following effects: atmospheric electricity, cosmic rays, and the solar wind. The contribution from atmospheric electricity arises from fine weather conduction through the atmosphere \((\sim +2000 \text{ A})\), point discharges, precipitation, and lightning. Despite considerable ambiguities in the measurement of the above it is safe to conclude\(^3\) \(5\)

\[
|I_{\text{atmospheric electricity}}| < 10^3 \text{ A}.
\]

(Pomansky's estimate of 200 A is perhaps over-enthusiastic.)

To estimate the effect of cosmic radiation, observe that primary cosmic radiation consists mainly of protons.\(^6\) The Earth's magnetic field will reflect protons with energies less than \(\sim 2.5 \text{ GeV} \); measurements of the cosmic-ray flux then show that approximately 4000 protons/m² sec arrive at the top of the atmosphere.\(^6\) This is a current of \(\frac{1}{3} \) A. Some of this current will not reach the surface, since the protons and their secondaries suffer energy loss via ionization. Particles stopped by this mechanism form ions and thus contribute to the fine weather conduction current (considered above under atmospheric electricity). For those cosmic-ray particles that do reach
the surface we have

$$|f_{\text{cosmic}}| < \frac{1}{2} \text{ A}.$$  

Finally, consider the solar wind. This is a neutral plasma consisting mainly of protons and electrons, flowing at a typical speed of 400 km/sec, and of typical density 10 particles/cm$^3$ (Ref. 7). Taking the radius of the magnetosphere to be \( \sim 10 \) Earth radii we see that the current delivered to the magnetopause is \( +8 \times 10^9 \) A (by protons), \( -8 \times 10^9 \) A (by electrons).

It is perhaps not obvious that the solar wind can be entirely neglected for the purposes of this technique. Observe that solar-wind particles have kinetic energies of order: \( T_p \sim 850 \text{ eV}, T_e \sim \frac{1}{2} \text{ eV} \). This is certainly very much less than the rigidity cutoff imposed by the Earth's magnetic field (\( E \sim 2.5 \text{ GeV} \) for protons or electrons), so most of the solar wind is deflected, and never makes it past the magnetopause. However, some solar-wind particles may penetrate the magnetopause, possibly be accelerated, and contribute to the (pseudo) trapped radiation contained in the magnetosphere. These particles have energies in the range: 1 eV \( \leq T_e \leq 3 \text{ MeV} \), and 1 eV \( \leq T_p \leq 300 \text{ MeV} \), though typically energies are of order 1-10 keV. When and if these particles hit the top of the atmosphere they will rapidly lose energy by ionization and be stopped within a distance

\[
\begin{align*}
T_p &= 300 \text{ MeV}, \quad R = 472 \text{ atm m} = 58 \text{ g cm}^{-2} \quad (\text{Ref. 10)}, \\
T_e &= 10 \text{ keV}, \quad R = 0.022 \text{ atm m} = 2.7 \times 10^{-5} \text{ g cm}^{-2} \quad (\text{Ref. 11)}, \\
T_e &= 3 \text{ MeV}, \quad R = 13.67 \text{ atm m} = 1.675 \text{ g cm}^{-2} \quad (\text{Ref. 12)}, \\
T_e &= 10 \text{ keV}, \quad R = 0.22 \text{ atm m} = 2.7 \times 10^{-4} \text{ g cm}^{-2} \quad (\text{Ref. 11}).
\end{align*}
\]

Since the thickness of the atmosphere is 1000 g cm$^{-2}$, it is clear that all such particles will be stopped long before reaching the surface of the Earth. In stopping they will produce ions, these ions contribute to the fine weather conduction current previously considered, and need concern us no further.

Assembling the above contributions yields

$$eN_e |\omega_e + (\Delta Q_p)\omega_p + (\Delta Q_n)\omega_n|$$

$$\leq (2 \times 10^{-5} + 10^3 + \frac{1}{2}) \text{ A},$$

leading to the constraint

$$|\omega_e + (\Delta Q_p)\omega_p + (\Delta Q_n)\omega_n|$$

$$\leq 3.5 \times 10^{-30} \text{ sec}^{-1} = 1.1 \times 10^{-22} \text{ yr}^{-1}.$$

\[
\langle \Delta Q_p \rangle \omega_p = \sum n \omega_p (\Delta Q = n) - \sum n \omega_p (\Delta Q = n, \Delta B = 0) + \sum n \omega_p (\Delta Q = n, \Delta B \neq 0) \\
= \langle \Delta Q_p (\Delta B = 0) \rangle \omega_p (\Delta B = 0) + \langle \Delta Q_p (\Delta B \neq 0) \rangle \omega_p (\Delta B \neq 0),
\]

and, using an analogous decomposition for neutrons, we see that

$$|\langle \Delta Q_p (\Delta B = 0) \rangle \omega_p (\Delta B = 0) + \langle \Delta Q_n (\Delta B = 0) \rangle \omega_n (\Delta B = 0)|$$

$$< 2 \times 10^{-22} + |\langle \Delta Q_p (\Delta B \neq 0) \rangle + \langle \Delta Q_n (\Delta B \neq 0) \rangle| \times 10^{-26} \text{ yr}^{-1}.$$

The second term is significant compared to the first only in the extremely unlikely event that \( |\langle \Delta Q_p (\Delta B \neq 0) \rangle + \langle \Delta Q_n (\Delta B \neq 0) \rangle| \geq 10^6 \); thus one may feel justified in stating

$$|\langle \Delta Q_p \rangle \omega_p + (\Delta Q_n)\omega_n|_{\Delta B = 0} < 2 \times 10^{-22} \text{ yr}^{-1}.$$

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**PARTICLE PHYSICS**

Consider the geophysical constraint

$$|\omega_e + (\Delta Q_p)\omega_p + (\Delta Q_n)\omega_n| < 1.1 \times 10^{-22} \text{ yr}^{-1}.$$  

Further information may be extracted by appealing to other experimental data. Note that Pomansky performed his original analysis assuming that nucleons are completely stable, thereby obtaining a limit on the electron lifetime. However, it is more reasonable to use the experimental limit, \( \omega_e < 5 \times 10^{-23} \text{ yr}^{-1}, \) to conclude that

$$|\langle \Delta Q_p \rangle \omega_p + (\Delta Q_n)\omega_n| < 2 \times 10^{-22} \text{ yr}^{-1}.$$  

The work of Steinberg and Evans enables us to place limits on the rate of baryon-nonconserving decays, specifically,

$$\omega_p (\Delta B = 0), \omega_n (\Delta B = 0) < 10^{-26} \text{ yr}^{-1}.$$  

Thus, observing that

$$|\langle \Delta Q_p (\Delta B = 0) \rangle \omega_p (\Delta B = 0) + \langle \Delta Q_n (\Delta B = 0) \rangle \omega_n (\Delta B = 0)|$$

$$< 2 \times 10^{-22} + |\langle \Delta Q_p (\Delta B \neq 0) \rangle + \langle \Delta Q_n (\Delta B \neq 0) \rangle| \times 10^{-26} \text{ yr}^{-1}.$$  

The second term is significant compared to the first only in the extremely unlikely event that \( |\langle \Delta Q_p (\Delta B \neq 0) \rangle + \langle \Delta Q_n (\Delta B \neq 0) \rangle| \geq 10^6 \); thus one may feel justified in stating

$$|\langle \Delta Q_p \rangle \omega_p + (\Delta Q_n)\omega_n|_{\Delta B = 0} < 2 \times 10^{-22} \text{ yr}^{-1}.$$
To proceed further, we must make some guesses concerning the branching ratios of the various \( \Delta B = 0, \Delta Q \neq 0 \) decays of the bound nucleon. For these modes kinematics limits the possible decay schemes to

\[
\begin{align*}
    n &\to p + (\text{electrons}) + (\text{neutrinos}) + (\text{photons}) , \\
p &\to n + (\text{electrons}) + (\text{neutrinos}) + (\text{photons}) .
\end{align*}
\]

The number of neutrinos produced is at most two or three depending on the precise nuclear masses involved. Looking at any table of isotopes one sees that the \( p \to n \) decay is almost always kinematically forbidden (as reflected in the rarity of \( \beta^+ \) decay as compared to \( \beta^- \) decay). Among the \( n \to p \) decays, the production of more than one electron would appear rather unlikely (as reflected in the extreme rarity of double-\( \beta \) decay), thus allowing us to estimate

\[
|\langle \Delta Q_p \rangle \omega_p + (\Delta Q_n) \omega_n|_{(\Delta B = 0)} \approx |\langle \Delta Q_p \rangle \omega_n|_{(\Delta B = 0)} = \sum_\lambda n \omega_\lambda (\Delta Q = n, \Delta B = 0)
\]

\[
= (+1) \times \omega(n \to p + \text{neutrals}) + (+2) \times \omega(n \to p + e^+ + \text{neutrals}) + (0) \times \omega(n \to p + e^- + \text{neutrals}) .
\]

Using the approximation considered above we may deduce

\[
\omega(n \to p + \text{neutrals}) + 2 \omega(n \to p + e^+ + \text{neutrals}) < 2 \times 10^{-22} \text{ yr}^{-1} .
\]

This now leads to the limits

\[
\begin{align*}
    \omega(n \to p + \text{neutrals}) &< 2 \times 10^{-22} \text{ yr}^{-1} , \\
    \omega(n \to p + e^+ + \text{neutrals}) &< 10^{-22} \text{ yr}^{-1} .
\end{align*}
\]

These limits should be compared with the best available, that of Barabanov et al.,\(^5\) who obtained

\[
\omega(n \to p + \text{neutrals}) < 4.4 \times 10^{-24} \text{ yr}^{-1} .
\]

Note that the limit of Barabanov et al., while 50 times stronger than the geophysical limit quoted above, holds only for neutrons bound in \(^{3}H\)Ga, whereas the geophysical limits are averaged over the composition of the Earth. The only way for these new limits to be seriously in error would be to have some \( \Delta Q < 0 \) channel (e.g., \( p \to n + \text{neutrals} \), or \( n \to p + e^+ + e^- + \text{neutrals} \)) present with a width comparable to the channels considered.

Significant improvement of these limits would require both a better experimental limit on the electron lifetime, and an improved understanding of atmospheric electricity. Improved theoretical arguments concerning \( \Delta B = 0, \Delta Q \neq 0 \) nucleon branching ratios would also be gratifying.

Finally, it is somewhat amusing to observe that charge conservation, which practically all theoreticians expect to hold exactly, is experimentally on a less secure footing than baryon conservation, which is now expected to be only approximate.

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\[\text{References}\]


4. E. E. McDonald, Sci. Am. 188 (No. 4), 32 (1953).


