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TOWARDS THE OBSERVATION OF HAWKING RADIATION IN BOSE–EINSTEIN CONDENSATES

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Acoustic analogues of black holes (dumb holes) are generated when a supersonic fluid flow entrains sound waves and forms a trapped region from which sound cannot escape. The surface of no return, the acoustic horizon, is qualitatively very similar to the event horizon of a general relativity black hole. In particular Hawking radiation (a thermal bath of *phonons* with temperature proportional to the "surface gravity") is expected to occur. In this note we consider quasi-one-dimensional supersonic flow of a Bose–Einstein condensate (BEC) in a Laval nozzle (converging-diverging nozzle), with a view to finding which experimental settings could magnify this effect and provide an observable signal. We discuss constraints and problems for our model and identify the issues that should be addressed in the near future in order to set up an experiment. In particular we identify an experimentally plausible configuration with a Hawking temperature of order 70 n K; to be contrasted with a condensation temperature of the order of 90 n K.

Keywords: Laval nozzle; converging-diverging nozzle; supersonic flow; acoustic horizon; surface gravity.

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1. Background

Whereas the Hawking radiation phenomenon is a cornerstone of black hole thermodynamics, and is believed to be central to developing a quantum theory of gravity, the phenomenon has never been seen either experimentally or observationally. This fact has led to increasing interest in trying to detect the effect indirectly, by simulating it in condensed matter systems. Acoustic analogues of black holes (dumb holes) are formed by supersonic fluid flow.[1,2] The flow entrains sound waves and forms a trapped region from which sound cannot escape. The surface of no return, the acoustic horizon, is qualitatively very similar to the event horizon of a general relativity black hole; in particular Hawking radiation (in this case a thermal bath of *phonons* with temperature proportional to the "surface gravity") is expected to occur.[1,2] There are at least three physical situations in which acoustic horizons are known to occur: Bondi–Hoyle accretion,[6] the Parker wind[7] (coronal outflow from a star), and supersonic wind tunnels. Recent improvements in the creation and control of BECs (see e.g. Ref. 3) have lead to a growing interest in these systems as experimental realizations of acoustic analogs of event horizons. Two key observations are that the speed of sound is very low in these systems and that the physical temperature of the condensate itself is *extremely* low. To quantify the expected size of the effect we consider supersonic flow of a BEC through a Laval nozzle (converging-diverging nozzle) in a quasi-one-dimensional approximation. We show that this geometry allows the existence of a fluid flow with acoustic horizons without requiring any special external acceleration mechanism, and we study this flow with a view to finding situations in which the Hawking effect is large. We present simple physical estimates for the "surface gravity" and Hawking temperature. A fluid flow in a Laval nozzle geometry has also been considered in Ref. 4, but in a different context: that paper deals with a classical effect, related to the Hawking effect, but does not consider the quantum physics of Hawking radiation itself.[5]

2. Laval Nozzle

A general problem with the realization of acoustic horizons is that most of the background fluid flows so far studied seem to require very special fine-tuned forms for the external potential used to actively accelerate the fluid (see e.g. the Schwarzschild-like geometry in Ref. 8). In this respect a possible improvement toward the realizability of acoustic horizons is the use of a trap which "geometrically constrains" the flow. An example of such a geometry is the so called Laval nozzle. [This does not necessarily imply physical contact between the fluid and the walls of the nozzle — strictly speaking, a hard wall makes no sense in BEC experiments; a geometrical constraint imposed by a passive potential is good enough to avoid fine-tuning issues.] In particular we shall consider a pair of Laval nozzles; this provides a system which includes a region of supersonic flow bounded between two subsonic regions, the same geometry used in building supersonic wind tunnels.

In engineering parlance the second Laval nozzle is called a "supersonic diffuser."[9] When initially setting up the supersonic flow the throat of the diffuser should be wider than that of the upstream Laval nozzle (so that it is the upstream nozzle that first "chokes" forcing the flow supersonic). Once the supersonic flow has been established the throat of the diffuser can be narrowed until it is just marginally wider than that of the upstream Laval nozzle. This enables the so-called "breakdown shock" at the diffuser to be made arbitrarily weak while maintaining hydrodynamic stability of the system.[9,10] Under normal operating conditions there is no shockwave at the upstream Laval nozzle where the flow first goes supersonic.

Consider such a nozzle pointing along the z axis. Let the cross sectional area be denoted $A(z)$. We apply, with appropriate modifications and simplifications, the calculation of Ref. 8. The crucial approximation is that transverse velocities (in the x and y directions) are small with respect to velocity along the z axis. Then, assuming steady flow, we can write the continuity equation in the form

$$\rho(z)A(z)v(z) = J\,; \qquad J = \text{constant}\,. \tag{1}$$

The Euler equation (including for the moment possible external body forces $d\Phi/dz$, and internal viscous friction f_v) reduces to

$$\rho v\frac{dv}{dz} = -\frac{dp}{dz} - \rho\,\frac{d\Phi}{dz} + f_v\,. \tag{2}$$

Finally, we assume a barotropic equation of state $\rho = \rho(p)$, and define $X' = dX/dz$. Then continuity implies

$$\rho' = -\rho\frac{(Av)'}{(Av)} = -\rho\left[\frac{A'}{A} + \frac{v'}{v}\right] = -\rho\left[\frac{A'}{A} + \frac{a}{v^2}\right], \tag{3}$$

while Euler implies

$$\rho a = -\frac{dp}{d\rho}\rho' - \rho\Phi' + f_v\,. \tag{4}$$

Defining the speed of sound by $c^2 = dp/d\rho$,[a]. and eliminating ρ' between these two equations yields a form of the well-known "nozzle equation"

$$a = -\frac{v^2}{c^2 - v^2}\left(c^2\left[\frac{A'}{A}\right] - \Phi' + \frac{f_v}{\rho}\right). \tag{5}$$

[a]Strictly speaking the speed of sound is not constant transversally because of the non-constancy of the density at the boundary of the trap. This could be seen as a problem for our one-dimensional approximation. However we can argue that in the case of large N and repulsive interactions, the density of the condensate becomes quite flat. The actual decrease from the average density n to zero happens in one healing length. So in our case if the ring section is at least a few healing lengths wide it is still meaningful to consider surfaces of constant c with radius smaller by one healing length than the ring cross-section. As long as the nozzle is more than one healing length wide, the density in the central region is almost constant along the transverse directions; this is a standard approximation in BEC literature and it is exactly what we need for our quasi-one-dimensional approximation

The presence of the factor $c^2 - v^2$ in the denominator is crucial and leads to several interesting physical effects. For instance, if the physical acceleration is to be finite at the acoustic horizon, we need

$$c^2\left[\frac{A'}{A}\right] - \Phi' + \frac{f_v}{\rho} \to 0\,. \tag{6}$$

This is a condition relating the shape of the nozzle to the external body force and specific friction. Experience with wind tunnels has shown that the flow will attempt to self-adjust (in particular, the location of the acoustic horizon will self-adjust) so as to satisfy this relation.

3. Free Flow

Let us now analyze the special case in which the external body force can be neglected $\Phi' = 0$ and no viscous friction is present $f_v = 0$. Whereas the latter condition is exactly satisfied in superfluid flow, the former can be viewed as a hard wall approximation for the Laval-nozzle trapping potential. This approximation is exact, $\Phi' = 0$, when considering quantities at the nozzle throat. Then the nozzle equation (5) reduces to

$$a = -\frac{c^2 v^2}{c^2 - v^2}\left[\frac{A'}{A}\right]. \tag{7}$$

Regularity now requires a "fine-tuning" condition

$$A' = 0 \tag{8}$$

at the horizon. That is, the acoustic horizon occurs at a point of minimum area; exactly the behavior which is physically seen in a Laval nozzle. (That a horizon cannot form at a maximum of the cross sectional area is established below.) Now apply the L'Hôpital rule at the horizon (using $A'|H = 0$)

$$a_H = \left.\frac{-c^4 A''/A}{(c^2)' - 2a_H}\right|_H. \tag{9}$$

We need to use

$$(c^2)' \equiv \frac{d^2p}{d\rho^2}\rho' = -\rho\frac{d^2p}{d\rho^2}\frac{(Av)'}{Av} \Rightarrow (c^2)'|_H = -\rho_H \left.\frac{d^2p}{d\rho^2}\right|_H \frac{a_H}{c_H^2}\,. \tag{10}$$

So that

$$a_H = \pm\frac{c^2}{\sqrt{2A}}\left.\sqrt{\frac{A''}{1 + (1/2)\rho[d^2p/d\rho^2]/c^2}}\right|_H. \tag{11}$$

It is extremely useful to consider the quantity

$$g = -\frac{1}{2}\frac{d(c^2 - v^2)}{dz}\,. \tag{12}$$

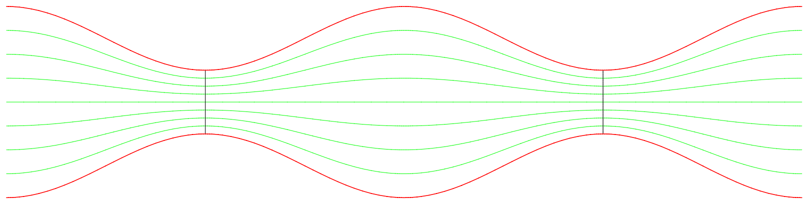

Fig. 1. A pair of Laval nozzles: The second constriction, a "supersonic diffuser," is used to bring the fluid flow back to subsonic velocities.

It is this combination g, rather than the physical acceleration of the fluid a, that more closely tracks the notion of black hole "surface gravity,"[2] and it is the limit of this quantity as one approaches the acoustic horizon that enters into the Hawking radiation calculation.[11] Since $g = a - (c^2)'/2$, the fine-tuning used to keep a finite at the acoustic horizon will also keep g finite there. In particular,

$$g_H = \left[1 + \frac{\rho}{2c^2}\frac{d^2p}{d\rho^2}\right] a_H \tag{13}$$

$$= \pm \frac{c_H^2}{\sqrt{2A_H}} \sqrt{1 + \frac{\rho}{2c^2}\frac{d^2p}{d\rho^2}}\bigg|_H \sqrt{A''_H}\,. \tag{14}$$

The first factor is of order c_H^2/R, with R the minimum radius of the nozzle, while the second and third factors are square roots of dimensionless numbers. This is in accord with our intuition based on dimensional analysis.[2,8] If $A'' < 0$, corresponding to a maximum of the cross section, then g_H is imaginary which means no event horizon can form there. The two signs $\pm$ correspond to either speeding up (black hole horizon) and slowing down (white hole horizon) as you cross the horizon, both of these must occur at a minimum of the cross sectional area $A'' > 0$ (see Fig. 1). If the nozzle has a circular cross section, then the quantity A''_H is related to the longitudinal radius of curvature R_c at the throat of the nozzle, in fact $A''_H = \pi R/R_c$.

4. Bose–Einstein Condensate

Equation (14) for the surface gravity of the acoustic horizon was derived considering an inviscid, irrotational, barotropic flow. Hence, in order to apply the previous discussion to the case of a BEC, we need to consider Bose–Einstein condensates for which the superfluid description of the condensate applies.[3] This regime is encountered when the Gross–Pitaeveskii (GP) equation holds and the quantum pressure can be neglected. Indeed, the GP equation is derived from a many-body Hamiltonian in the case in which most of the atoms are in the condensed phase (mean field approximation) and the gas is very dilute in such a way that only the two-body short-range collisions are relevant. (This is equivalent to imposing $n|a|^3 \ll 1$, where n is the number density and a is the scattering length for bi-atomic interactions.) Once the GP holds the BEC can be described as a superfluid (inviscid and irrotational) with a barotropic equation of state $p \propto n^2$ with the only "anomaly" being the presence of the quantum potential term $V_{\rm Q} = \hbar^2\nabla^2\sqrt{n}/(2m\sqrt{n})$. This quantum potential is generally neglected in the case of strong repulsive interaction when the density profile becomes smooth. The requirement that this assumption holds not only for the background configuration but also for perturbations around it, implies that one must restrict attention to those perturbations with wavelengths long with respect to the "healing length" $\xi = (4\pi a n)^{-1/2}$ which is the typical scale over which the quantum potential is of the same order of the interaction term in the GP equation.[3] We shall consider later the consequences of relaxing this last approximation.

The speed of sound in a uniform BEC is (corrections due to the finite size of the trap can be shown to be marginal)

$$c = \frac{2\hbar}{m}\sqrt{\pi a n}\,, \tag{15}$$

so we have, rather simply

$$g_H = \pm\frac{c_H^2}{\sqrt{A_H}}\sqrt{3A''_H/4}\,. \tag{16}$$

This implies, at a black hole horizon [future horizon], a Hawking temperature[1,2,11]

$$k_{\rm B}T_H = \frac{\hbar g_H}{2\pi c_H} = \hbar\frac{c_H}{2\pi\sqrt{A_H}}\sqrt{\frac{3A''_H}{4}}\,. \tag{17}$$

Ignoring for now the issue of gray-body factors (they are a refinement on the Hawking effect, not really an essential part of the physics), the phonon spectrum peaks at

$$\lambda_{\rm peak} = 4\pi^2\sqrt{A_H}\sqrt{\frac{4}{3A''_H}} = 4\pi^2\sqrt{\frac{4}{3}RR_c}\,. \tag{18}$$

This extremely simple result relates the Hawking emission to the physical size of the constriction and a factor depending on the flare-out at the narrowest point. Note that you cannot permit A''_H to become large, since then you would violate the quasi-one-dimensional approximation for the fluid flow that we have been using in this note. (There is of course nothing physically wrong with violating the quasi-one-dimensional approximation, it just means the analysis becomes more complicated.) The preceding argument suggests strongly that the best we can realistically hope for is that the spectrum peaks at wavelength $\lambda_{\rm peak} \gtrsim \sqrt{A_H} \approx R$. Note that this is the analog, in the context of acoustic black holes, of the fact that the Hawking flux from general relativity black holes is expected to peak at wavelengths near the physical diameter of the black hole — up to numerical factors depending on charge and angular momentum. You can (in principle) try to adjust the equation of state to make the second factor in (14) larger, but this is unlikely to be technologically feasible.

In closing this analysis of the Hawking radiation for a wind-tunnel-shaped BEC flow, we revisit the issue of the neglect of the quantum potential. How is our analysis changed regarding the physical realizability of the flow, and the nature of the Hawking radiation, if the quantum potential is taken into account? We can summarize the most important changes in three points:

The "fine tuning" condition (8) is replaced by a more general relation which includes the quantum potential and hence higher-order derivatives in the density at the horizon. For this reason the generalization of (8) cannot be seen as a special fine-tuning condition. Indeed, this implies that the horizon will no longer occur exactly at the minimum diameter of the nozzle, and that divergence of the acceleration at the horizon will no longer be generic. Basically the quantum potential can be seen to play a role similar to that of a viscous term (which instead introduces higher-order derivatives in the speed of the flow) as described in Ref. 8.

Thanks to the presence of the quantum potential shockwaves are no longer a generic feature. In fact, shockwaves are generic features of inviscid systems. (Viscosity gives a finite thickness to physical, as opposed to idealized, shocks.) The quantum potential comes into play when gradients in the density are on scales comparable to the healing length, effectively avoiding discontinuities in the hydrodynamical quantities. It is nevertheless important to consider configurations where large gradients are naturally avoided (like the wind tunnel considered here) because gradients on scales of the order of the healing length would be problematic for the mean field approximation (and hence the GP equation) to hold.

Finally one has to take into account that the quantum potential is directly related to the existence of the quartic term in the dispersion relation for perturbations in the BEC

$$\omega^2 = c^2\left(k^2 + \frac{k^4}{K_0^2}\right), \tag{19}$$

where $K_0 \propto \xi^{-1}$.[16,17] This is the so-called Bogoliubov dispersion relation[3] which is a phonon spectrum modified at high momenta in such a way as to recover the "infinite" propagation speed typical of the non-relativistic system we started with (the non-linear Schödinger equation, the GP equation). This high-momentum modification of the BEC quasi-particle dispersion relation can be important to the generation of Hawking radiation given that any outgoing Hawking quanta with finite wavelength far away from the horizon will be blue-shifted when traced back toward the dumb hole and eventually will reach a point where its wavelength will be of the order of the healing length. This type of modification of the dispersion relation has been extensively considered in relation to the possible resolution of the so called trans-Planckian problem. Fortunately it is known, thanks to model calculations in field theories with explicit high-momentum cutoffs, that the low energy physics of the emitted radiation is largely insensitive to the nature and specific features of the cutoff if the peak wavelength of the Hawking flux is much larger than the cut-off scale (in our case the healing length). We shall explicitly check that this condition is satisfied for our system in the following section. As a final remark we stress that "superluminal" dispersion relations of the Bogoliubov kind can still have a potentially dangerous side. This is related to an intrinsic instability in the quantum particle creation when compact ergoregions (regions where the timelike Killing vector becomes spacelike) are present, a phenomenon christened "back hole laser."[18] This instability is relevant for system like the one considered in this letter. We shall see that instead of being a potential problem, this instability might be exploited as a possible venue for the detection of quantum particle creation in a BEC flow.

5. Size of the Effect

It is the fact that the peak wavelength of the Hawking radiation is of order the speed of sound (which in BEC systems is very low) that makes the effect so difficult to detect. Simultaneously one can easily realize that the minimal radius of the nozzle is quite constrained. The peak wavelength (18), a purely geometrical quantity, has to be much larger than the healing length for the hydrodynamic approximation to hold. We have also argued that for the quasi-one-dimensional approximation to hold the transverse size of the throat must be more than one healing length, in contrast an upper limit on the transverse size of the nozzle throat is provided by the requirement of stability of our system to vortex nucleation. The minimum diameter for a vortex in a BEC is several healing lengths. To obtain significant vortex production one would need a throat that was many healing lengths in diameter. Furthermore in a Laval nozzle the pressure is maximum at the throat, further suppressing the possibility of vortex nucleation. Thus the transverse dimensions of the throat should be several healing lengths. (In contrast, lengthwise the system can be many healing lengths long; which permits us to use the one-dimensional hydrodynamic approximation.) An exact calculation of the constraints on the ratio of healing length ξ to nozzle radius R is basically impossible, being determined by several experimental as well as theoretical issues. We shall here assume that $R = \chi\xi$ where χ is some positive number of order one or greater, $\chi \gtrsim 1$. Similarly, regarding the steepness of the throat we can write $R_c = \alpha^2 R$ where α is a constant equal to or greater than one: $\alpha \gtrsim 1$. Taking into account these definitions one can see that

$$\lambda_{\rm peak} = 45\chi\alpha\xi \gtrsim 45\xi\,. \tag{20}$$

We then see that in our model even for the "extremal" values $\chi = \alpha = 1$ the peak wavelength of the Hawking radiation is always larger than the healing length and that even in the case in which χ is fixed by the constraint of vortex formation one can tune the shape of the nozzle parametrized by α is such a way to be in a regime where $\lambda \gg \xi$.

This shows that in this particular model the bulk of the Hawking flux exists in a regime which is definitely "sub-Planckian", given that the healing length plays in BEC analogue model the same role that the Planck scale plays in semiclassical gravity. Our equation (20) also implies that our model is a specific counter-example to Unruh's recent objection to the viability of BEC systems for studies of analog Hawking radiation.[12]

Another possible concern regarding the size of $\lambda_{\rm peak}$ is that our analysis also requires $\lambda_{\rm peak} \ll L$ where L is the typical size of the condensate. In typical traps this size is about 100 times the healing length (10 microns of size against 10^{-1} microns for the healing length) leaving a small region for the validity of $\xi \ll \lambda_{peak} \ll L$. In our case L would be the total length of the tunnel and one can hope that this length can be extended in order to be in the range of 10–100 microns. This is in any case a technical issue rather than a fundamental obstruction to the realization of such a configuration.

In BECs experiments it is common to have a sound speed of order 6 mm/s, then, considering a throat $R = 1$ micron (that is, $\chi = 10$) and $R_c = R$ (that is, $\alpha = 1$) we obtain a Hawking temperature $T_H \approx 7$n K. Comparing this T_H to the condensation critical temperature

$$T_{\rm condensate} \approx 90 \text{ n K}\,, \tag{21}$$

we see that in this situation the Hawking effect, although tiny, is at least comparable in magnitude to other relevant temperature scales. In order to enhance T_H one could increase the density of the condensate, this simultaneously increases the speed of sound and allows for smaller R by reducing ξ. A similar enhancement can be achieved by increasing the scattering length. In particular, recent experiments indicate that this quantity can be tuned by making use of the so called Feshbach resonance[13] and that increments by a factor up to 100 can be experimentally obtained.[14] The propagation speed (15) could thereby be enhanced by a factor up to 10. Basically, as long as the dilute gas limit holds, it appears possible to enhance n or a in order to improve the Hawking temperature by at least a factor 10:

$$T_H \approx 70 \text{ n K}\,. \tag{22}$$

This places us much closer to the condensation temperature. If the Hawking effect can be experimentally realized in these situations, it may be sufficiently large to disrupt the condensate configuration, or even the condensate itself. The power loss due to Hawking radiation would be

$$P = \sigma T_H^4 A_H = \frac{3\hbar c_H^4}{5120\pi^2 A_H}(A_H'')^2\,. \tag{23}$$

Numerically (even including the effect of the Feshbach resonance), this is extremely small $P \approx 10^{-48}$ W. Despite this, it is still possible that a detection of the quantum particle creation can be obtained via some other kind of instability. Let us elaborate this point: The Garay *et al.* analysis[15] shows (even without assuming the hydrodynamic approximation) that it should be possible to create stable BEC configurations with acoustic horizons. In their particular analysis, in which a BEC is spinning in a effectively one-dimensional ring, they found stable configurations surrounded by unstable regions. But the Laval nozzle geometry being analyzed can be seen as a specific implementation of the ring configuration if we feed the outflow from the second (diffuser) nozzle back into the inflow of the first upstream nozzle. Once the system is engineered to be in a configuration which would be stable in the absence of the Hawking effect, one can search for the purely quantum effect of Hawking emission.

An alternative route is to look at the instability regions (see e.g. the analysis regarding the nature of the instability regions in Garay *et al.*[15]). As previously discussed, because of the high-frequency "superluminal" dispersion and the presence of the two horizons in our system one should encounter instabilities related to the "black hole laser" mechanism[18]). There are good reasons to believe that the instability bands found by Garay *et al.* are due to this effect, and that the existence of stability regions is due to the periodic boundary conditions imposed by the ring geometry. In this case one can have additional hopes of detecting a signal of quantum particle creation by looking at the instability regions. In fact it is expected that such quantum phonon creation is driven to a run-away production of phonons[18] (phonon lasing) which should be much easier to detect than the Hawking flux because of its typical spectrum and because it would lead to a new type of instability, disrupting the otherwise classically stable configuration. It is nevertheless important to stress here that the laser instability is classical in nature and that the laser mechanism can excite existing classical fluctuations (real phonons) or create them from the vacuum. In the first case one would expect excitation of particles in coherent states, in the second one should have squeezed states. Correlation measurements on phonons on different sides of the horizon could provide a way to distinguish these effects although it is possible that the classical effect would be overwhelming with respect to the quantum one.

Finally we describe a third possibility for an indirect detection of a Hawking flux, based on the measurement of the slow-down of the flow on long timescales (several Hawking cycles) due to energy loss induced by the Hawking radiation. The problems in this case are related to the technical possibility of suppressing or distinguishing any other competitive dissipative process, and to the actual obtainable lifetime of a stable condensate.

6. Conclusions

The present note complements the analysis by Garay *et al.*[15] in that it provides a rationale for simple physical estimates of the analog Hawking effect. Additionally, the current study identifies several specific physical mechanisms by which the Hawking temperature can be manipulated: via the speed of sound, the nozzle radius, the equation of state, and the degree of flare-out at the throat. In particular, we have identified a plausible configuration with a Hawking temperature of order 70 n K; to be contrasted with the critical condensation temperature which is of the order of 90 n K (keeping in mind that the actual physical temperature of the condensate is estimated to be well below this value). We have discussed the experimental constraints on our model and checked that there is not, at least at a theoretical level, any *a priori* obstruction to considering such a system as a plausible set up for the laboratory reproduction of the Hawking phenomenon. Finally we have discussed several alternative signatures for the relevant quantum particle production.

While there is no serious dispute that the Hawking effect will actually occur in general relativistic black holes, there is currently no direct evidence to this effect. It is for this reason that analogue models are so important — they may provide an indirect window on fundamental aspects of curved space quantum field theory.

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References

1. W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981); *Phys. Rev.* **D51**, 2827 (1995).
2. M. Visser, *Class. Quantum Grav.* **15**, 1767 (1998).
3. F. Dalfovo, S. Giorgini, L.P. Pitaeveskii and S. Stringari, *Rev. Mod. Phys.* **71**, 463 (1999).
4. M. Sakagami and A. Ohashi, *Prog. Theor. Phys.* **107**, 1267 (2002).
5. M. Nouri-Zonoz and T. Padmanabhan, “The classical essence of black hole radiation,” gr-qc/9812088.
6. H. Bondi, *Mon. Not. Roy. Astron. Soc.* **112**, 195 (1952).
7. E. N. Parker, *Astrophys. J.* **143**, 32 (1966).
8. S. Liberati, S. Sonego and M. Visser, *Class. Quantum Grav.* **17**, 2903 (2000).
9. A. H. Shapiro, *Dynamics and Thermodynamics of Compressible Fluid Flow* (Ronald Press, 1953), Vol. 1, Chaps. 4 and 5.
10. W. F. Hughes and J. A. Brighton, *Fluid Dynamics* (McGraw-Hill, 1999), see Chap. 7.
11. M. Visser, *Int. J. Mod. Phys.* **D12**, 649 (2003).
12. W. Unruh, “Measurability of dumb hole radiation?,” in *Artificial Black Holes*, eds. M. Novello, M. Visser and G. Volovik, (World Scientific, Singapore, 2002); Workshop on “Analog Models of General Relativity” (Rio de Janeiro, October, 2000); http://mcs.vuw.ac.nz/~visser/Analog
13. J. L. Roberts, N. R. Claussen, S. L. Cornish and C. E. Wieman, *Phys. Rev. Lett.* **85**, 728 (2000).
14. S. L. Cornish, N. R. Claussen, J. L. Roberts, E. A. Cornell and C. E. Wieman, *Phys. Rev. Lett.* **85**, 1795 (2000).
15. L. J. Garay, J. R. Anglin, J. I. Cirac and P. Zoller, *Phys. Rev. Lett.* **85**, 4643 (2000); *Phys. Rev.* **A63**, 023611 (2001).
16. C. Barceló, S. Liberati and M. Visser, *Class. Quantum Grav.* **18**, 1137 (2001).
17. M. Visser, C. Barceló and S. Liberati, “Acoustics in Bose-Einstein condensates as an example of broken Lorentz symmetry,” hep-th/0109033.
18. S. Corley and T. Jacobson, *Phys. Rev.* **D59**, 124011 (1999).