We give a proof that there are incomparable Turing degrees.

Lemma 1. $\neg$CH *implies the existence of incomparable Turing degrees.*

Proof. We know that there are only countably many Turing machines, so for every Turing degree $\mathbf{d}$, there are only countably many degrees $\mathbf{a} \leqslant \mathbf{d}$. Suppose that the Turing degrees are linearly ordered; then their cofinality is at most ω_1, which now implies that there are $\aleph_1$ Turing degrees. We know that there are continuum many Turing degrees so if the degrees are linearly ordered then CH holds. □

Now let $\mathbb{P}$ be any notion of forcing such that $\Vdash_{\mathbb{P}} \neg\mathrm{CH}$. Let ψ be the sentence which states that there are two incomparable Turing degrees. The lemma implies that $\Vdash_{\mathbb{P}} \psi$. However, stated in second order arithmetic we see that ψ is Σ^1_1 (as Turing reducibility is arithmetically definable). Thus ψ is absolute between models of set theory. It follows that ψ holds.