The multiple deaths of Palatini f(R) gravity

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Motivation

- Theoretical: Clash between GR and QFT
  - * intrinsic limits, conceptual clash, ...
  - * Quantum corrections, string theory...
  - * higher order theories, Lorentz or EP violations...
- Observational: Inability to explain cosmological/astrophysical riddles without dark matter/energy
  - * 4% baryons, 20% dark matter, 76% dark energy!
  - * acceleration, deceleration, then acceleration
  - * cosmological constant and coincidence problems

Proposed way out

- Alternative theory of gravity which:
  - ☑ comes as a low energy limit of a more fundamental theory
  - ☑ includes ultraviolet/infrared corrections with respect to General Relativity
  - ☑ can account for some or all the unexplained observations

f(R) gravity as a toy theory

But we don't know the fundamental theory!

* *we could blame it on others...*

* *cook up toy theories and maybe even give them feedback - sounds much better!*

Typical Example: f(R) gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \quad \rightarrow \quad S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R)$$

Review: T. P. S. and Valerio Faraoni, arXiv:0805.1726 [gr-qc], commissioned by Rev. Mod. Phys.

Three versions of f(R) gravity

1. **Metric:** *variation only wrt the metric*

2. **Palatini:** *variation wrt metric and connection*

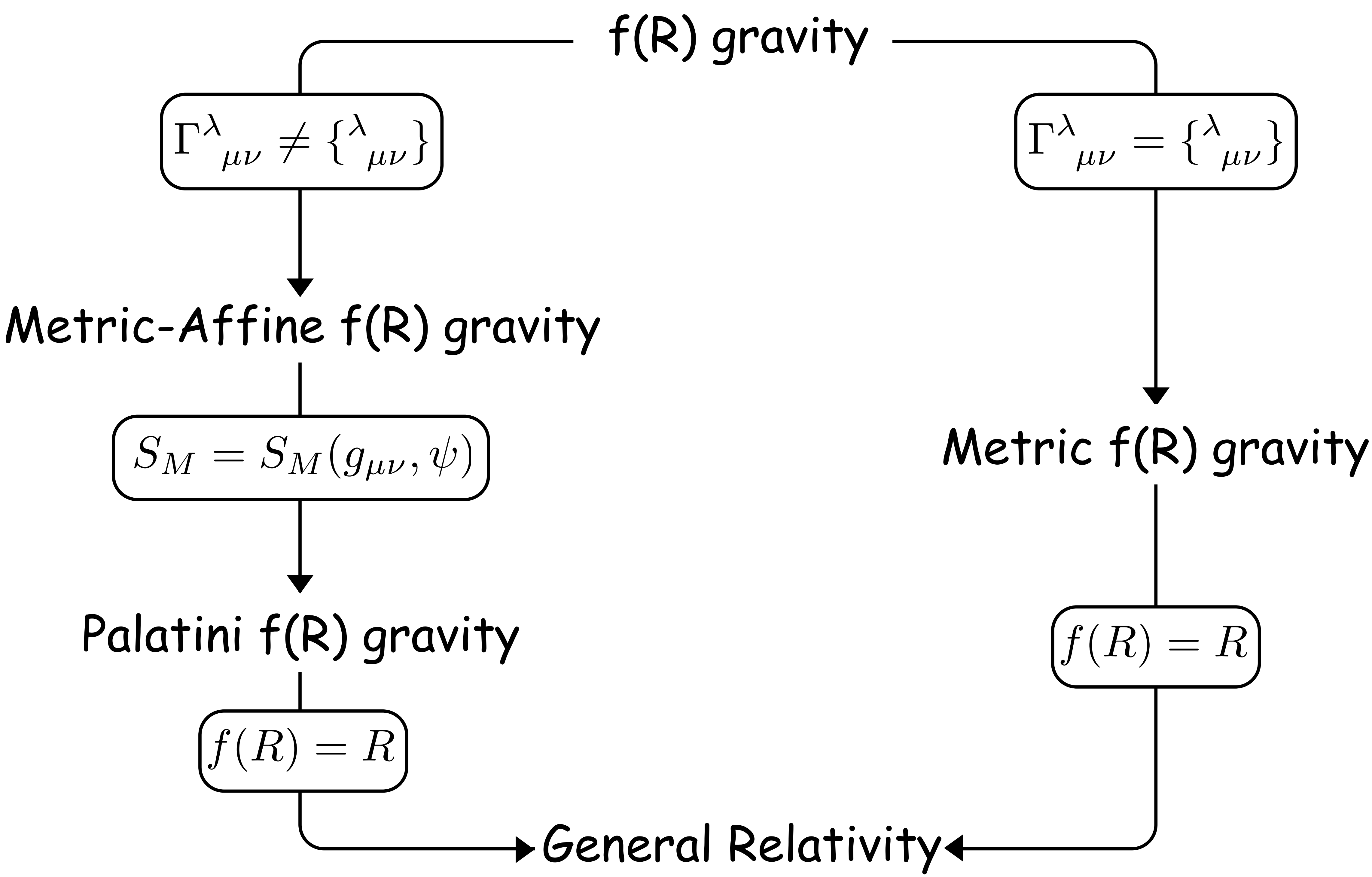
   * *the connection is an independent variable but does not enter the matter action!*

3. **Metric-affine:** *variation wrt metric and connection*

   * *the connection is an independent variable and enters the matter action*

Metric and Palatini variations both lead to GR for Einstein-Hilbert action (textbook)

Classification



Palatini f(R) gravity

Field equations:

$$
\begin{aligned}
f'(\mathcal{R})\mathcal{R}_{\mu\nu} - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} &= 8\pi G T_{\mu\nu} \\
\bar{\nabla}_\lambda \left(\sqrt{-g} f'(\mathcal{R}) g^{\mu\nu}\right) &= 0
\end{aligned}
$$

Trace of 1st field eq.:

$$
f'(\mathcal{R})\mathcal{R} - 2f(\mathcal{R}) = 8\pi G T
$$

Solving for the connection:

$$
\Gamma^\lambda_{\mu\nu} = \frac{g^{\lambda\sigma}}{f'(\mathcal{R})}\left[\partial_\mu \left(f'(\mathcal{R}) g_{\nu\sigma}\right) + \partial_\nu \left(f'(\mathcal{R}) g_{\mu\sigma}\right) - \partial_\sigma \left(f'(\mathcal{R}) g_{\mu\nu}\right)\right]
$$

Palatini f(R) gravity

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Algebraic equation!

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Algebraic equation!

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Solving for the connection:

Function of T

$$\Gamma^\lambda_{\mu\nu} = \frac{g^{\lambda\sigma}}{f'(\mathcal{R})}\left[\partial_\mu\left(f'(\mathcal{R})g_{\nu\sigma}\right) + \partial_\nu\left(f'(\mathcal{R})g_{\mu\sigma}\right) - \partial_\sigma\left(f'(\mathcal{R})g_{\mu\nu}\right)\right]$$

Equivalence with Brans-Dicke Theory

Starting with

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}f(\mathcal{R})$$

introducing an auxiliary scalar field yields

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(f(\chi)+f'(\chi)(\mathcal{R}-\chi)\right)$$

where variation gives

$$f''(\chi)(R-\chi)=0$$

I.e. Dynamically equivalent actions

Equivalence with Brans-Dicke Theory

Introducing the variables

$$\phi = f'(\chi), \qquad V(\phi) = \chi(\phi)\phi - f(\chi(\phi))$$

the action takes the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(\phi\mathcal{R} - V(\phi))$$

Using the field equations

$$\mathcal{R} = R + \frac{3}{2(f'(\mathcal{R}))^2}\left(\nabla_\mu f'(\mathcal{R})\right)\left(\nabla^\mu f'(\mathcal{R})\right) + \frac{3}{f'(\mathcal{R})}\Box f'(\mathcal{R})$$

Equivalence with Brans-Dicke Theory

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Careful!

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Equivalence with Brans-Dicke Theory

Outcome:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\phi R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

- ☑ Palatini f(R) gravity is equivalent to $\omega_0=-3/2$ Brans-Dicke Theory!

Field equation for the scalar:

$$(2\omega_0+3)+2V-\phi V' = 8\pi G T$$

The scalar field is not dynamical!

PPN limit and metric

Remarkable result*: *whether the theory has the correct Newtonian limit depends on the density!*

PPN metric:

$$h_{00}(t,x) = 2G_{\text{eff}}\frac{M_\odot}{r} + \frac{V_0}{6\phi_0}r^2 + \Omega(\rho)$$

Algebraic dependence on the matter!

* G. J. Olmo, Phys. Rev. D **72**, 083505 (2005); T. P. S., Gen. Rel. Grav. **38**, 1407 (2006)

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Conflict with Particle Physics

Non-perturbative corrections and strong coupling in the matter sector at low energies! *

- consider some matter field, e.g. the Higgs
- the connection is an auxiliary field
- Perturbative treatment breaks down
- non-minimal couplings between matter and metric though the connection!

* E. E. Flanagan, Phys. Rev. Lett. **92**, 071101 (2004); A. Iglesias *et al*., Phys. Rev. D **76** 104001 (2007)

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Singularities on stellar surfaces

Surface singularities in spherically symmetric polytropes for 3/2< Γ <2 ! EoS: $p = k\rho_0^\Gamma$ *

- ☑ *Unique exterior solution*
- ☑ *Matching with any interior leads to singularity*

Polytropes are restricted but...

- *no physically meaningful solution for isentropic gas or degenerate non-relativistic gas, Γ=5/3*
- *the problem is not restricted to polytropes*

* E. Barausse, T. P. S. and J. C. Miller, Class. Quant. Grav. **25**, 062001 (2008) (Fast Track); Class. Quant. Grav. **25**, 105008 (2008)

Non-cumulativity: the root of all evil

Field equations after eliminating the connection:

$$G_{\mu\nu} = \frac{8\pi G}{f'}T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left(\mathcal{R} - \frac{f}{f'}\right) + \frac{1}{f'}\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)f'$$
$$-\frac{3}{2}\frac{1}{f'^2}\left[\left(\nabla_\mu f'\right)\left(\nabla_\nu f'\right) - \frac{1}{2}g_{\mu\nu}\left(\nabla f'\right)^2\right]$$

Functions of T

- second order in the metric - higher order in the matter fields!
- matter enters the gravitational action through the back door leading to aforementioned issues

Conclusions and Outlook

- Astrophysics, Cosmology, Theoretical Physics all provide constraints for gravity theories

...but also...

- Toy theories of gravity can teach us a lot about the gravitational interaction

Several difficulties and subtleties in modified gravity:

- *Many constraints to satisfy*
- *Many directions to take*
- *A long way to go...*