

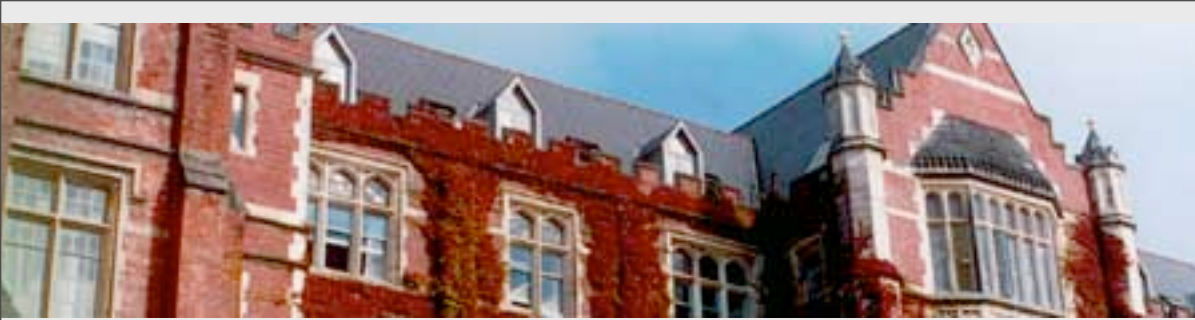
# Hubble, Hubble, toil and trouble...

Matt Visser

Royal Society Hudson lecture

23 May 2007



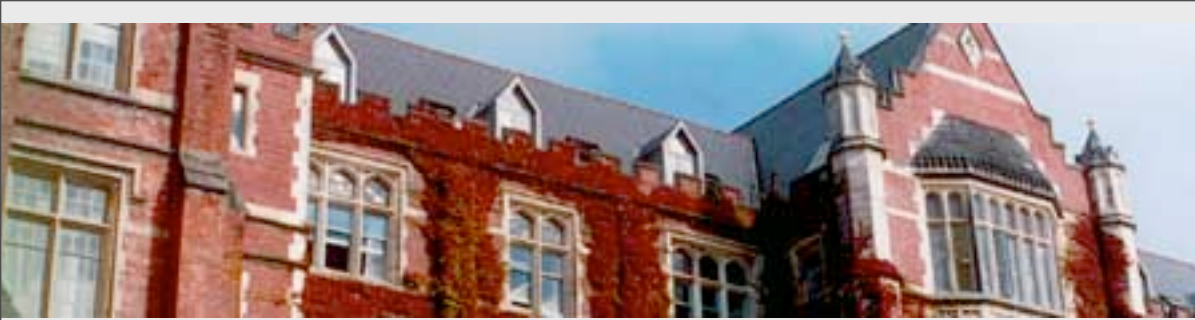


## Abstract:

The Hubble law describes the recession of the galaxies and the expansion of the universe --- it is one of the foundation stones of 20th century cosmology.

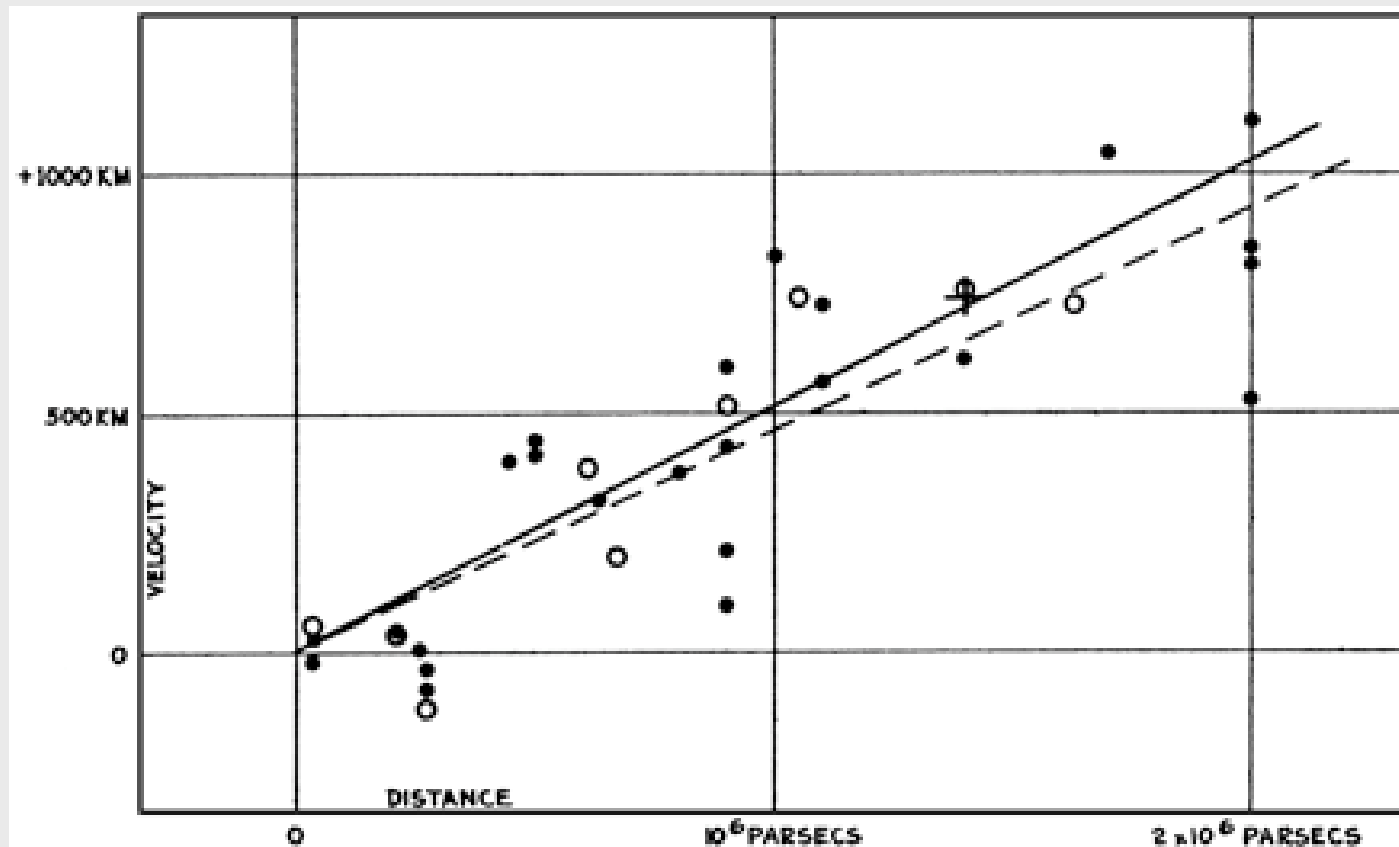
In particular, subtle deviations from the naive linear Hubble law underlie recent claims that the expansion of the universe is accelerating.

I will present a broad overview of the current situation, and possible lessons for the future.



# The original Hubble law:

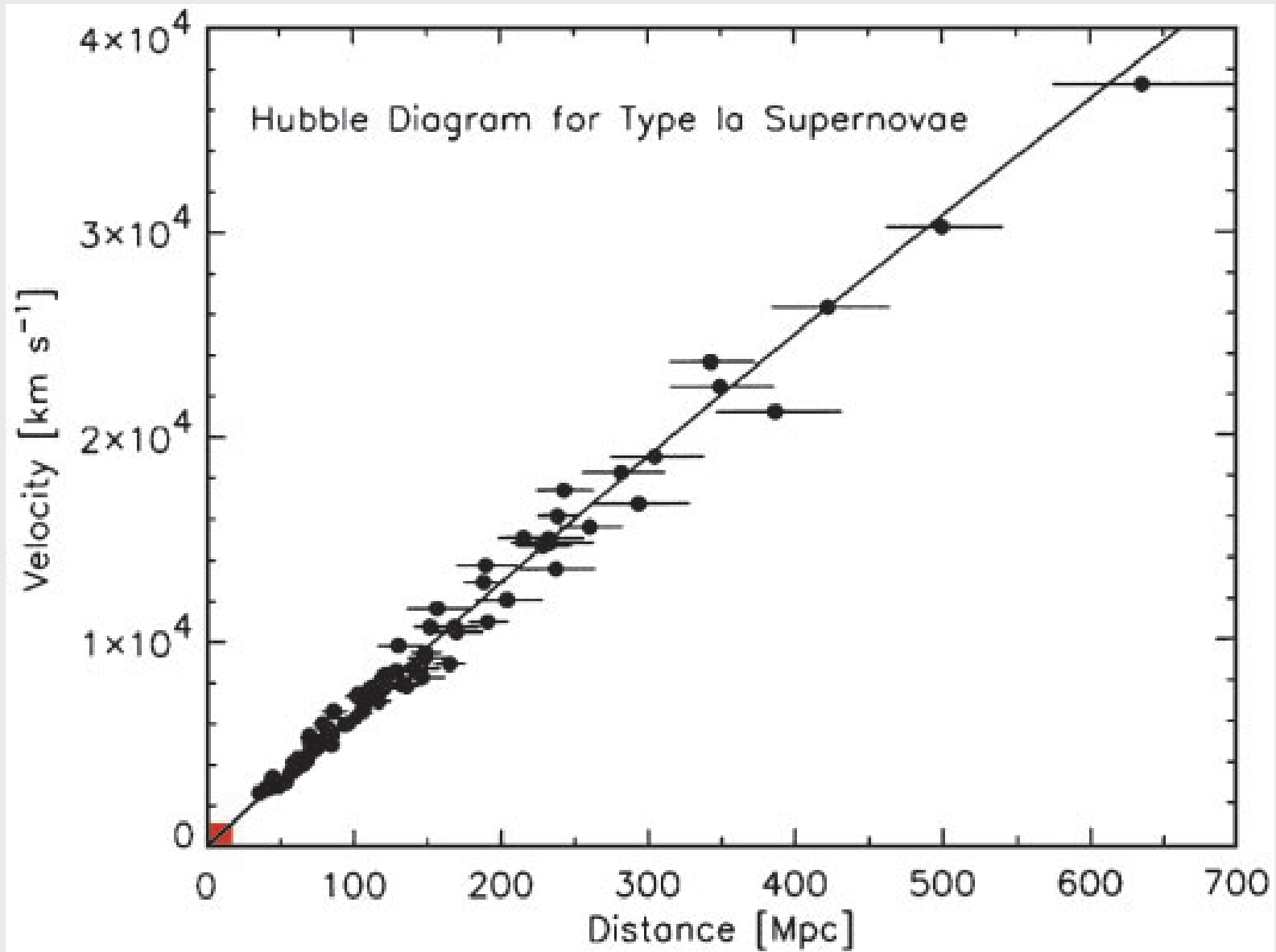
$$v = H_0 d; \quad H_0 \approx 500 \text{ (km/sec)/Mpc.}$$



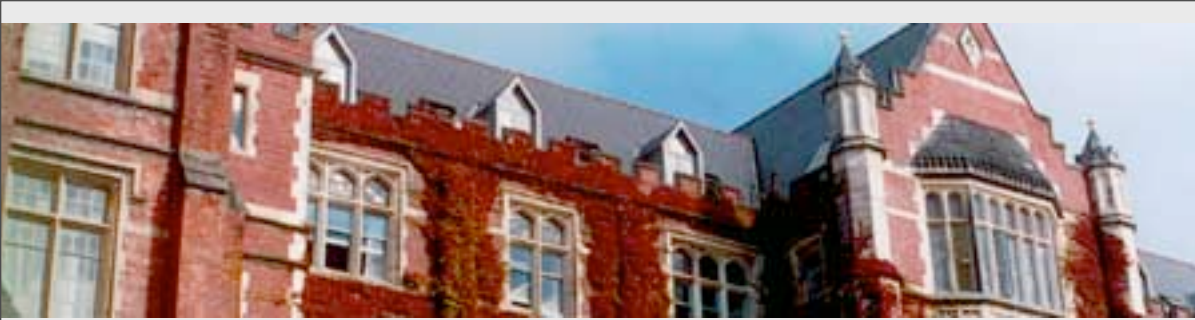
1929

[Hubble, E. P. (1929) *Proc. Natl. Acad. Sci. USA* 15, 168–173]

2003



The Hubble diagram for type Ia supernovae. The scatter about the line corresponds to statistical distance errors of  $< 10\%$  per object. The small red region in the lower left marks the span of Hubble's original Hubble diagram from 1929. [Kirshner 2003]



Modern  
Hubble law:

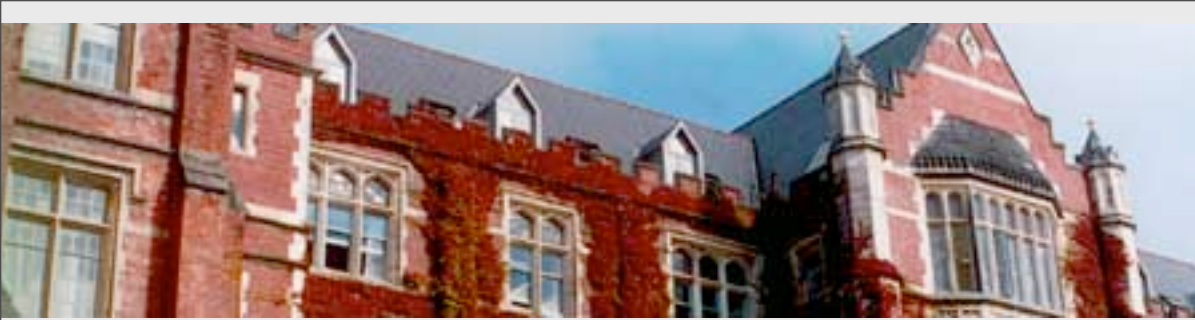
Prefer to work with redshift:

$$d = \frac{c z}{H_0} + \mathcal{O}(z^2).$$

$$1 + z = \frac{\lambda_{\text{received}}}{\lambda_{\text{emitted}}} = \frac{\omega_{\text{emitted}}}{\omega_{\text{received}}}.$$

Redshift is “easy” to measure...

distance is **extremely difficult** to measure...



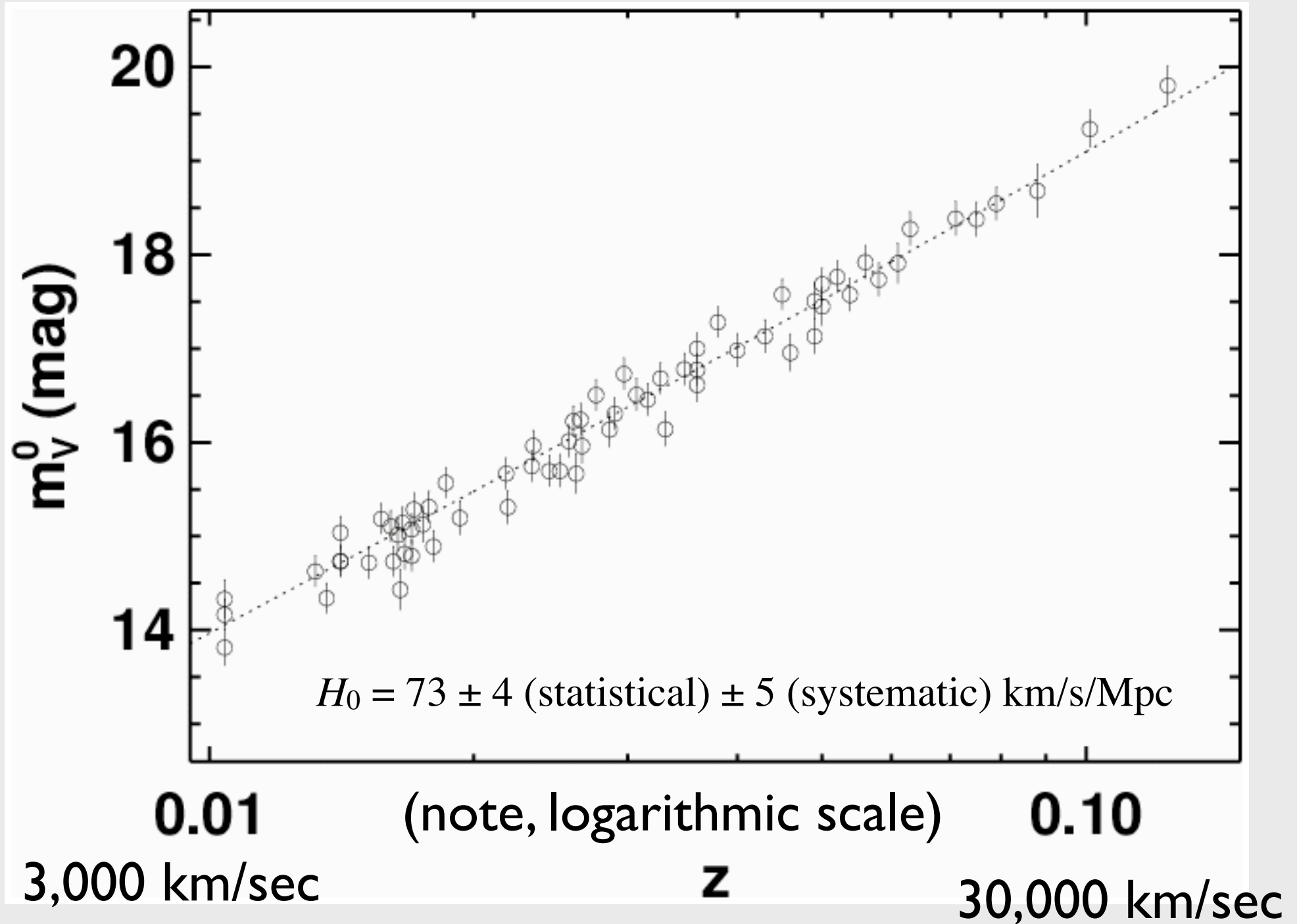
$$\mathcal{O}(z^2) ?$$

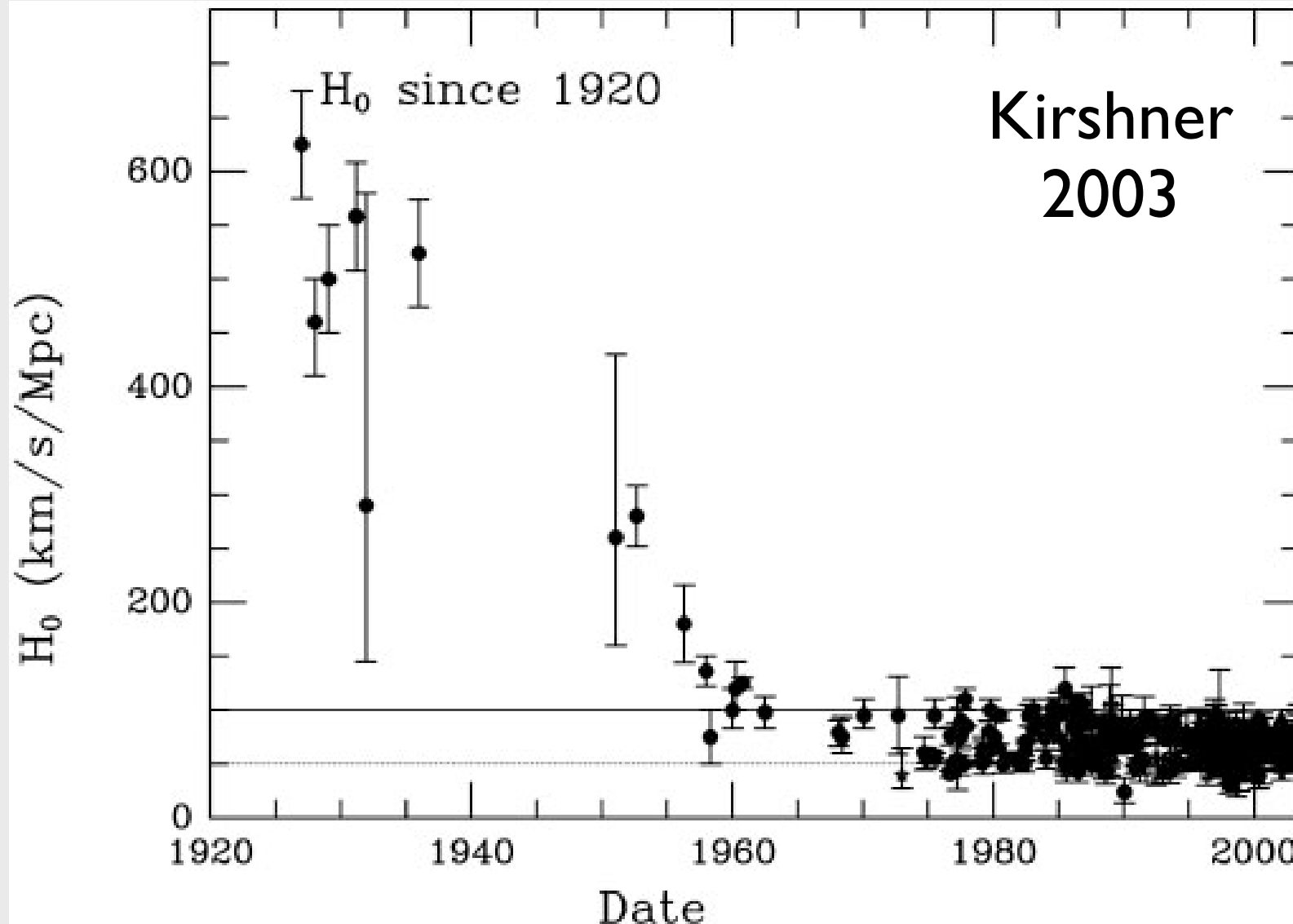
What's with the  $\mathcal{O}(z^2)$  ?

As the universe expands, one might reasonably expect the expansion to slow down...

The expansion is after all fighting against gravity...

So as you look further out into the night sky, since you are also looking further back in time, you might quite reasonably expect to be looking back to a time when the expansion might be faster than it is today.





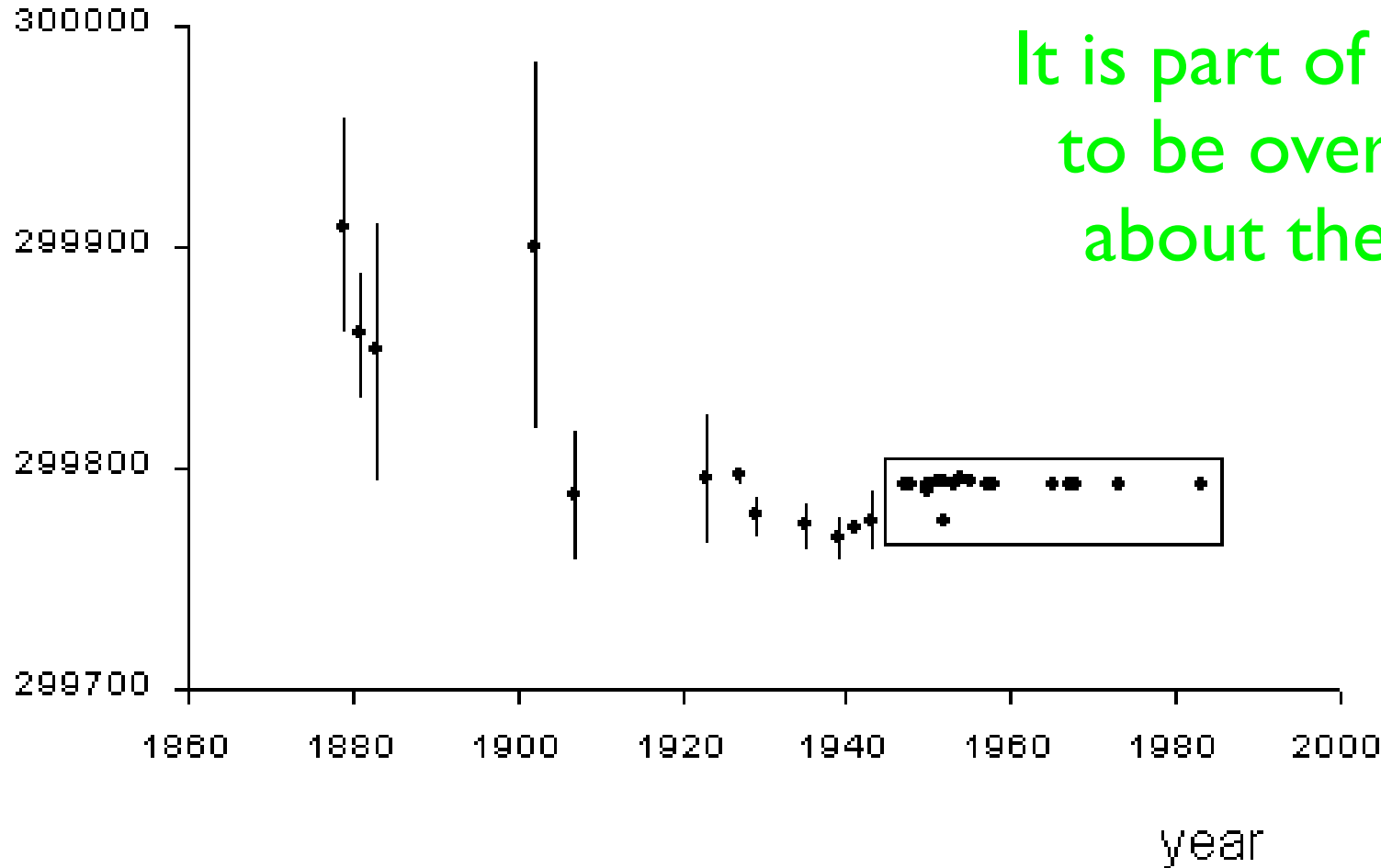
Published values of the Hubble parameter versus time. At each epoch, the estimated error in the Hubble parameter is small compared with the subsequent changes in its value. This is a symptom of underestimated systematic errors.



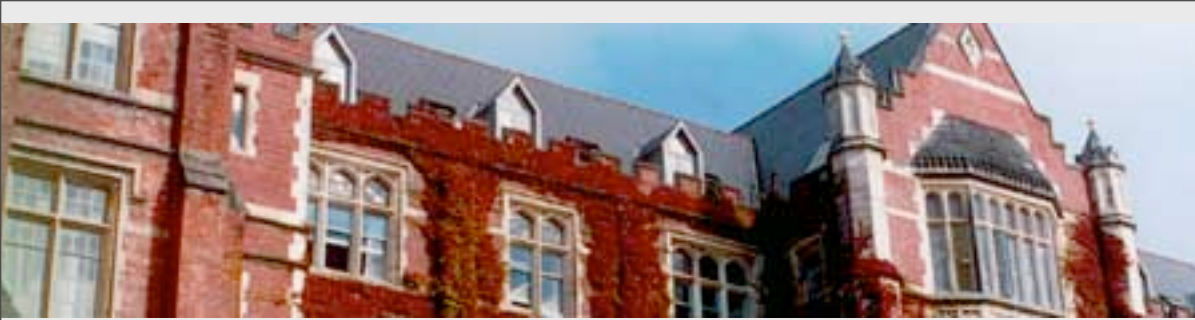


# Speed of light since 1880:

Speed (km/s)

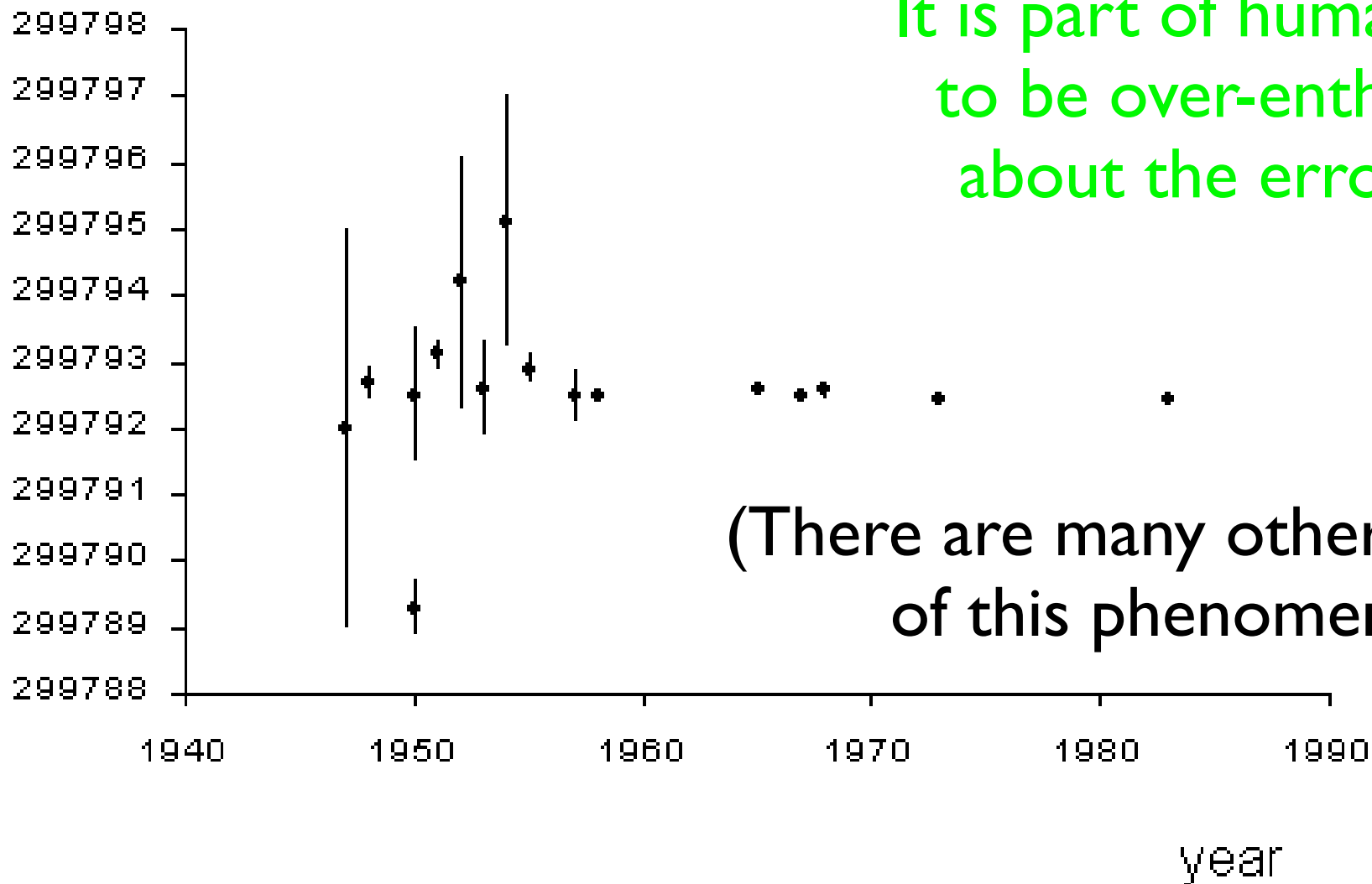


It is part of human nature  
to be over-enthusiastic  
about the error bars.



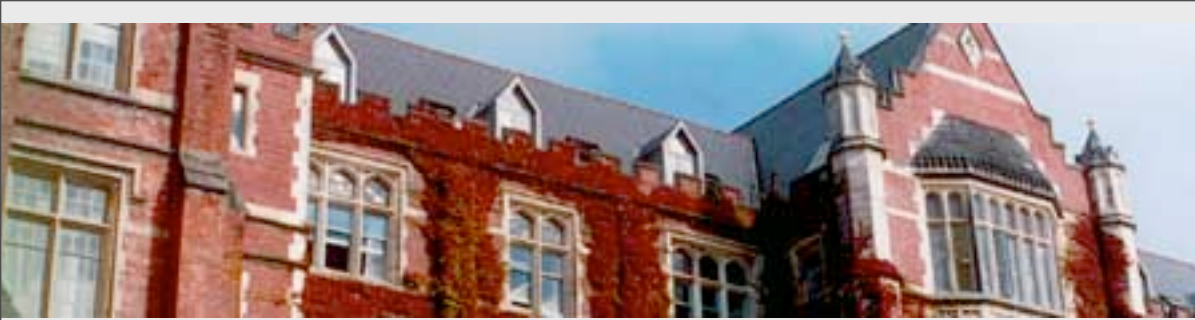
# Speed of light since 1940:

Speed (km/s)



It is part of human nature  
to be over-enthusiastic  
about the error bars.

(There are many other examples  
of this phenomenon.)



# Textbook cosmology:

[for example: Weinberg, Peebles]

FLRW universe:

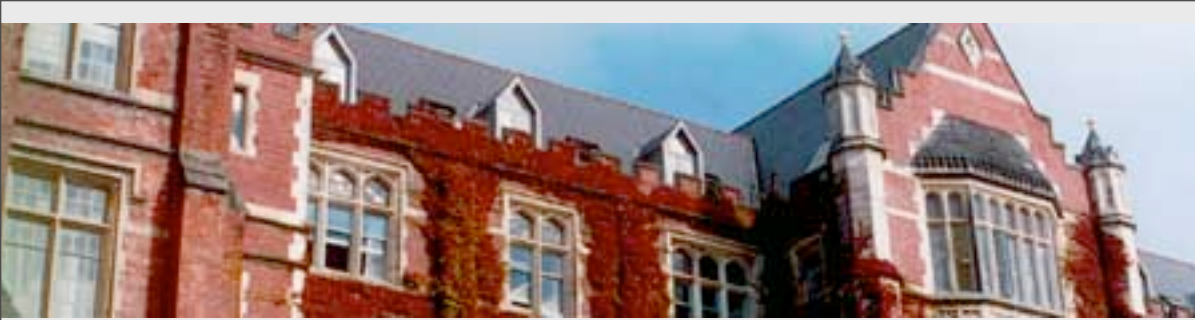
$$ds^2 = -c^2 dt^2 + a(t)^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 [d\theta^2 + \sin^2 \theta d\phi^2] \right\}$$

$a(t)$  is the “scale factor” of the universe; units of distance.

$r$  is just a label, dimensionless.

$$k \in \{-1, 0, +1\}.$$

To do this you just need symmetries,  
no dynamical assumptions...



## Definitions:

$$H(t) = \frac{\dot{a}(t)}{a(t)}; \quad \text{Hubble parameter.}$$

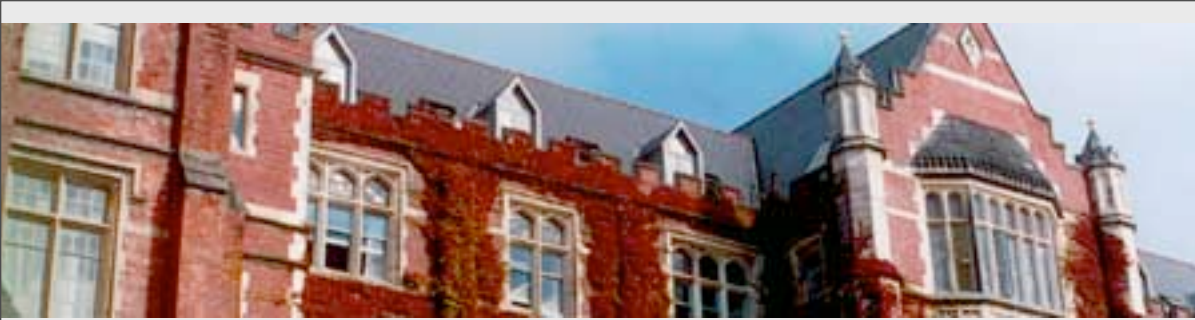
$$q(t) = -\frac{\ddot{a}(t) a(t)}{\dot{a}(t)^2}; \quad \text{dimensionless deceleration parameter.}$$

Until about 10 years ago everyone was expecting:

$$q(t) > 0; \quad \Leftrightarrow \quad \ddot{a} < 0.$$

Current data seems to suggest the opposite:

$$q(t) < 0; \quad \Leftrightarrow \quad \ddot{a} > 0. \quad ???$$



## Textbook exercise:

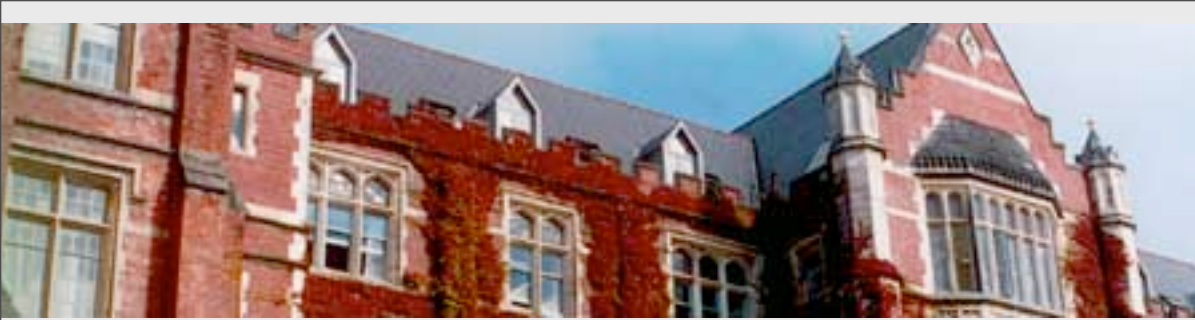
[for example: Weinberg, Peebles]

For a suitable definition of distance:

$$d_L(z) = \frac{c z}{H_0} \left\{ 1 + \frac{1}{2} [1 - q_0] z + O(z^2) \right\}.$$

$$H_0 = \frac{\dot{a}(t_0)}{a(t_0)}; \quad q(t_0) = -\frac{\ddot{a}(t_0) a(t_0)}{\dot{a}(t_0)^2}.$$

That is, evaluate the Hubble and deceleration parameters now (current epoch).



## Research project:

[for example: Chiba, Sahni, Visser]

$$d_L(z) = \frac{c z}{H_0} \left\{ 1 + \frac{1}{2} [1 - q_0] z - \frac{1}{6} \left[ 1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2 a_0^2} \right] z^2 + O(z^3) \right\}.$$

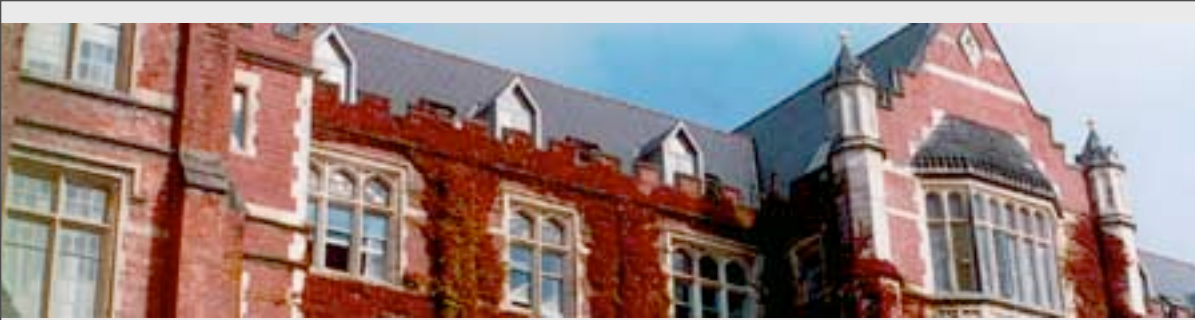
“Jerk” parameter:

$$j(t) = \frac{\ddot{a}(t) a(t)^2}{\dot{a}(t)^3}; \quad j_0 = \frac{\ddot{a}(t_0) a(t_0)^2}{\dot{a}(t_0)^3}.$$

Higher-order expansions are possible... [Visser]



Mechanics	Cosmology
position	scale factor
velocity	Hubble parameter
acceleration	deceleration
jerk	jerk parameter
snap	snap parameter
crackle	...
pop	...



## Modern tests:

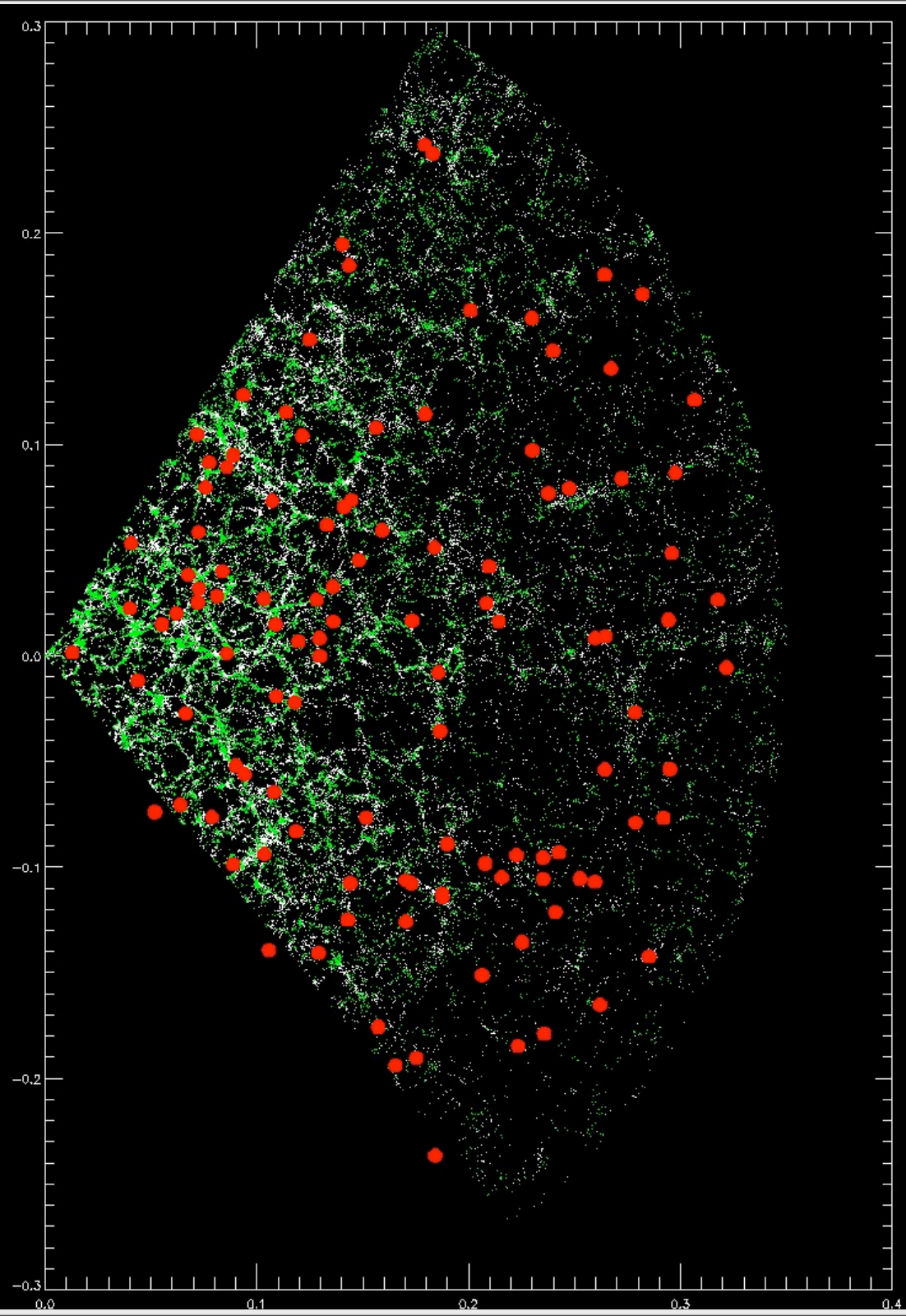
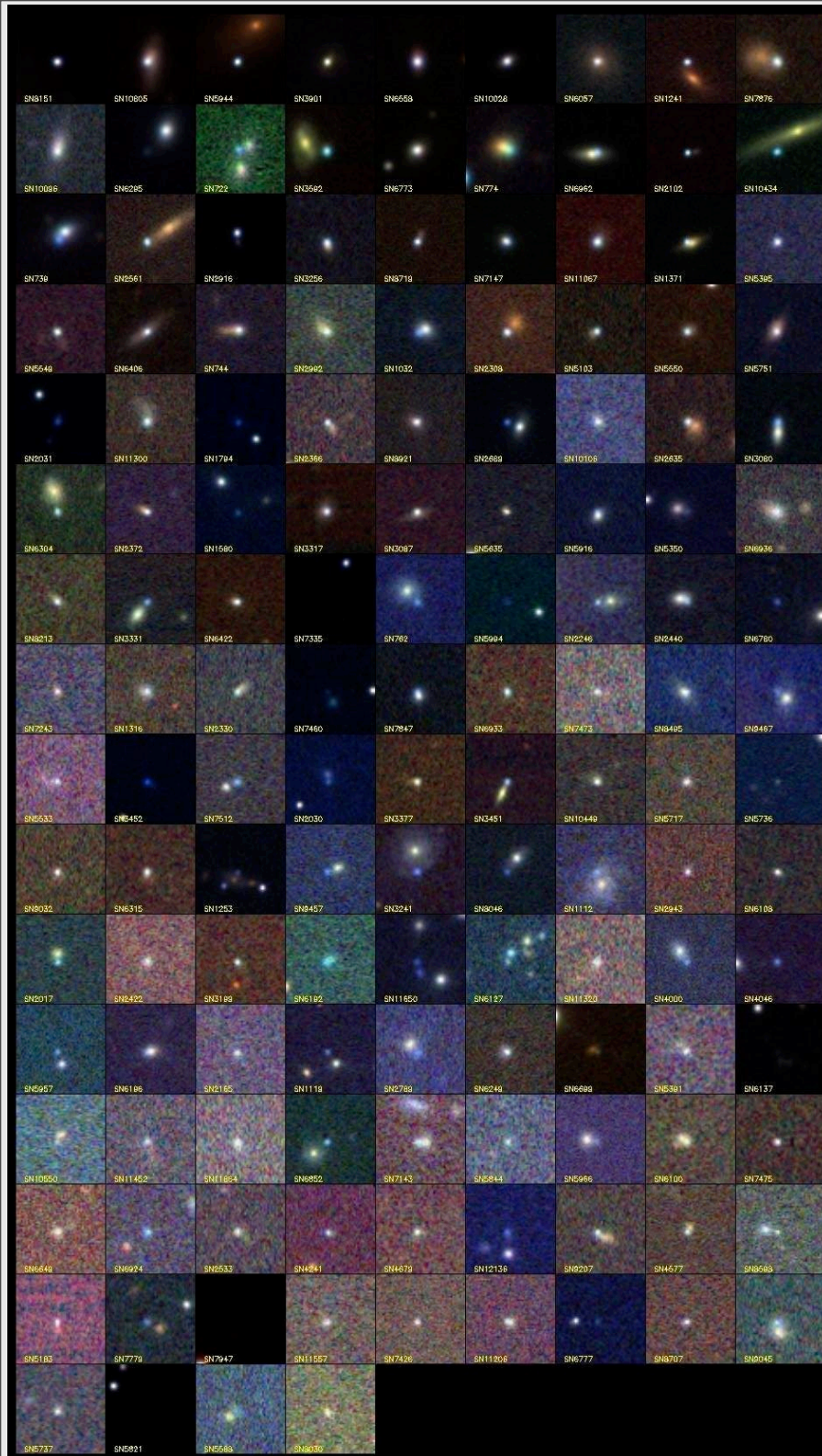
Latest tests of the Hubble law are based largely on **supernova data**, approximately 200 supernovae.

Now have data out to redshift:  $z \sim 1.75$

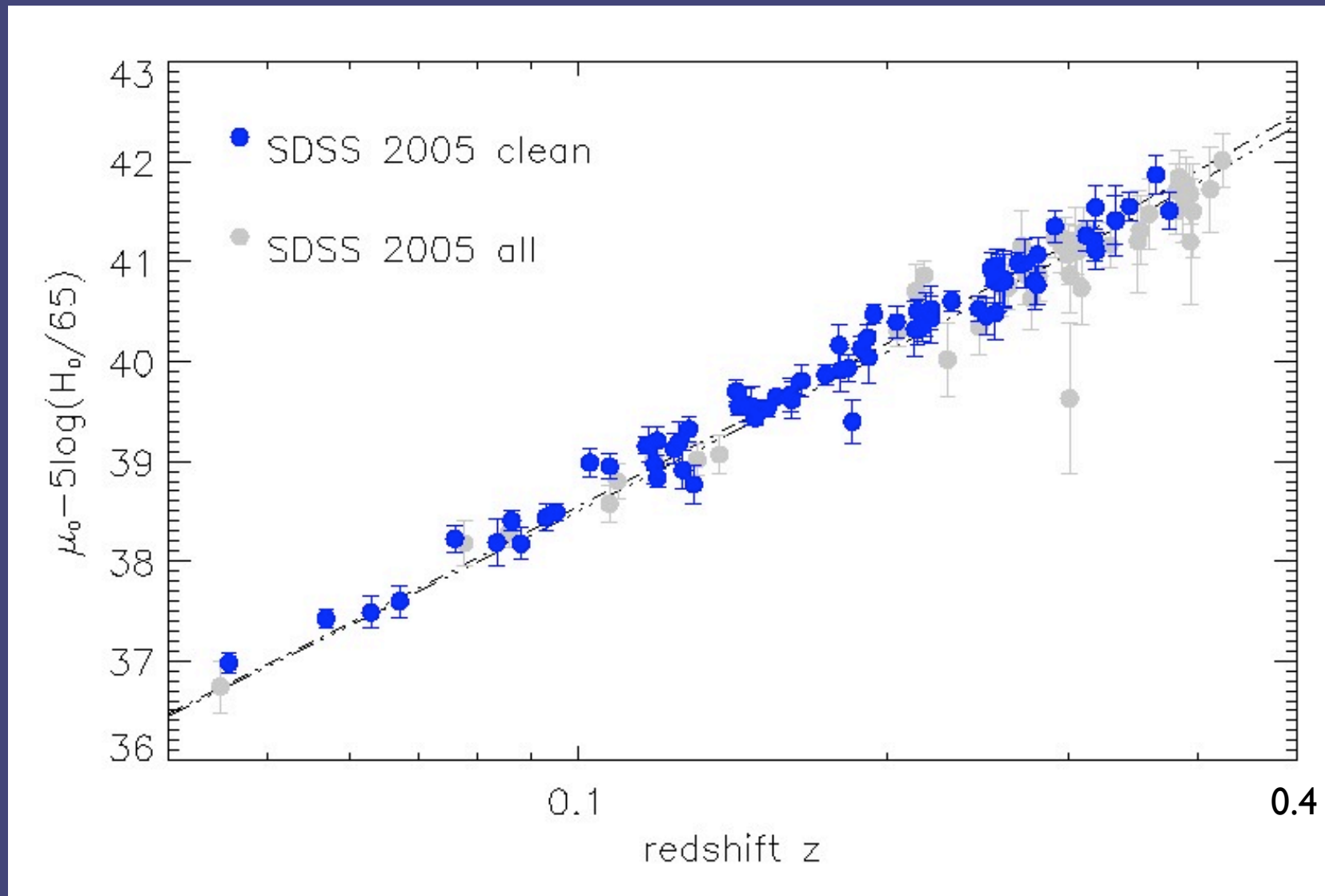
Major datasets: {  
Gold+Silver+Nearby (gold06)  
Supernova Legacy Survey (legacy05)

Lots of little “quirks” hiding in the processed data.



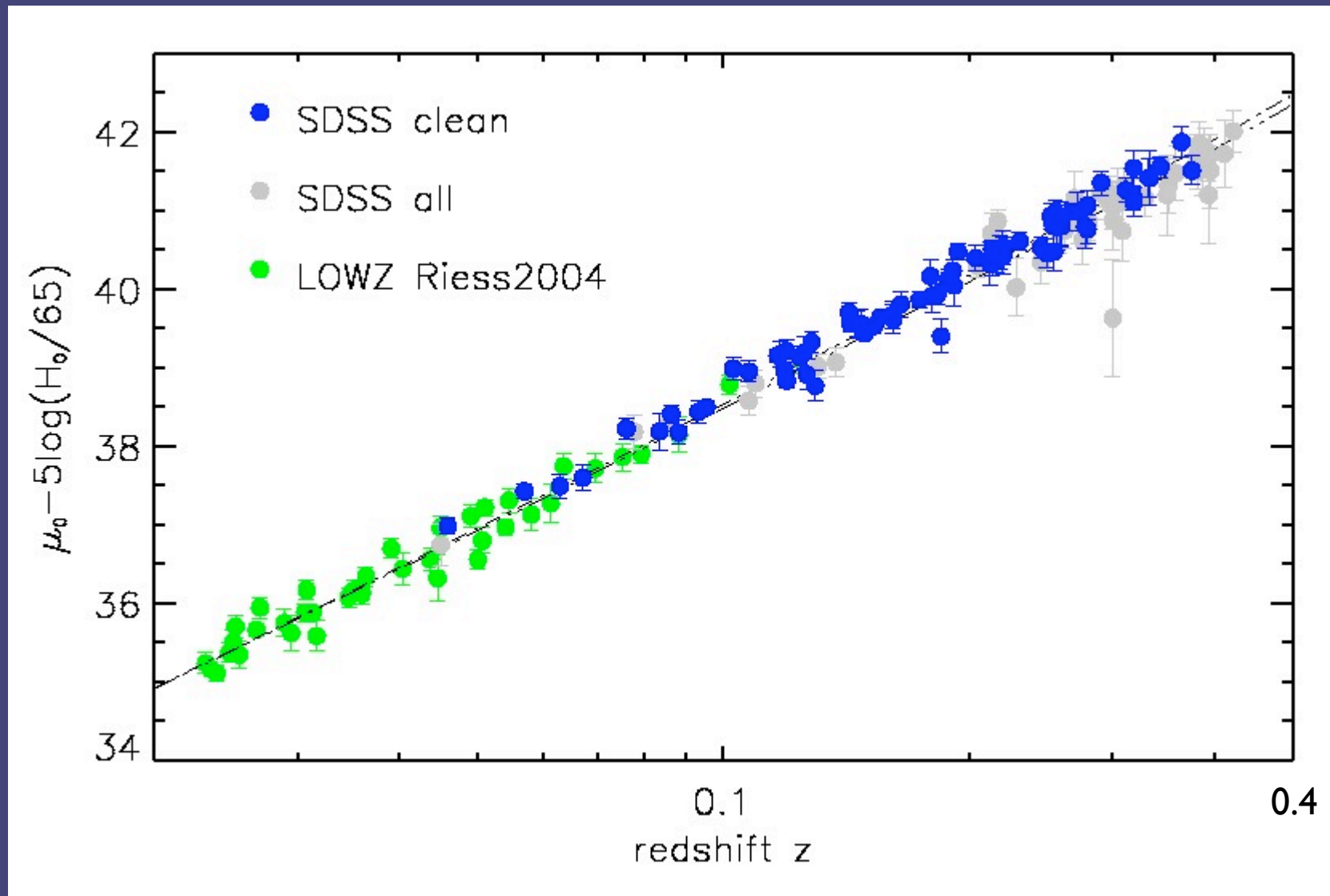


# Hubble Diagram (I)

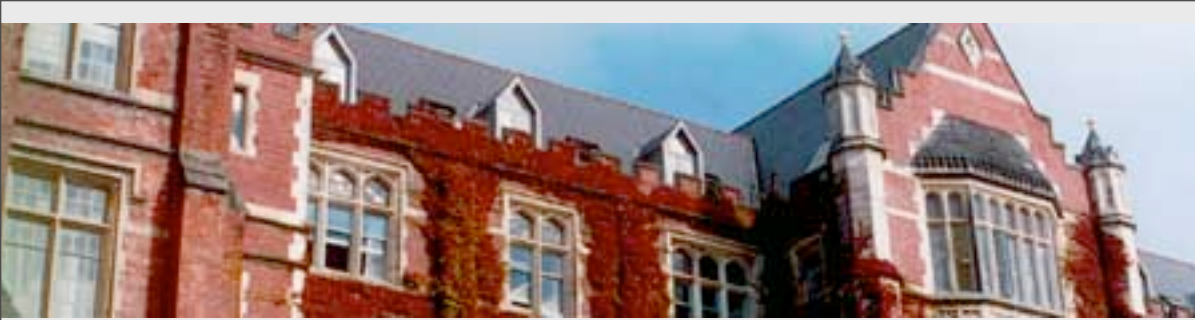


log-log

# Hubble Diagram (II)



log-log



## Distance scales:

Apart from the “**luminosity distance**”, one can consider:

- The “photon flux distance”:

$$d_F = \frac{d_L}{(1+z)^{1/2}}.$$

- The “photon count distance”:

$$d_P = \frac{d_L}{(1+z)}.$$

- The “deceleration distance”:

$$d_Q = \frac{d_L}{(1+z)^{3/2}}.$$

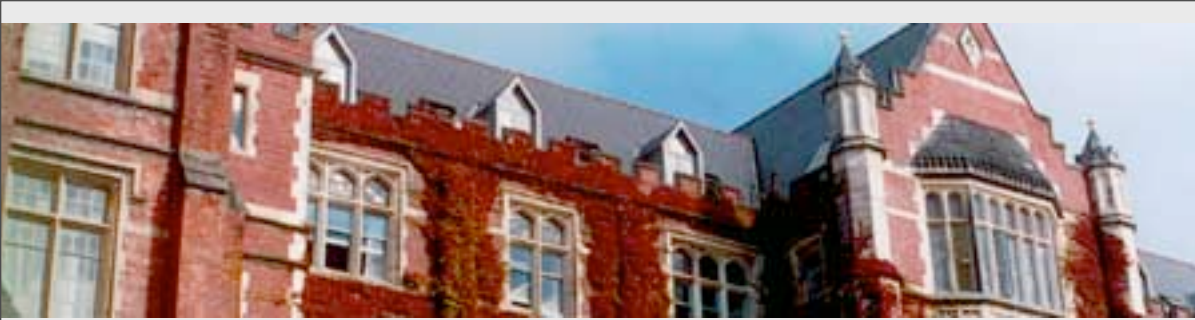
- The “angular diameter distance”:

$$d_A = \frac{d_L}{(1+z)^2}.$$

- The “distance modulus”:

$$\mu_D = 5 \log_{10}[d_L/(10 \text{ pc})] = 5 \log_{10}[d_L/(1 \text{ Mpc})] + 25.$$

**Celine Cattoen**  
**Matt Visser**  
**gr-qc/0703122**



## Distance scales:

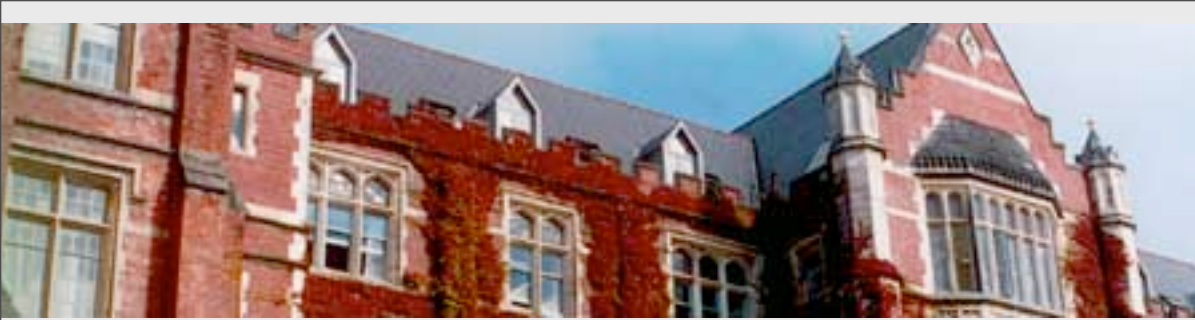
$$d_L \geq d_F \geq d_P \geq d_Q \geq d_A. \quad [\text{gr-qc}/0703122]$$

Overall scale: “Hubble distance”:  $d_H = \frac{c}{H_0},$

Best estimate:

$$H_0 = 73^{+3}_{-4} \text{ (km/sec)/Mpc}$$

$$d_H = 4100^{+240}_{-160} \text{ Mpc.}$$



Omega  
parameter:

$$\Omega_0 = 1 + \frac{kc^2}{H_0^2 a_0^2} = 1 + \frac{k d_H^2}{a_0^2}.$$

If we use the Einstein equations this is the ratio of total density to “Hubble density”...

--- but we do not need to use the Einstein equations, not in this lecture anyway...

(This lets one draw conclusions that are independent of whether or not the Einstein equations are valid.)



## Many Hubble laws:

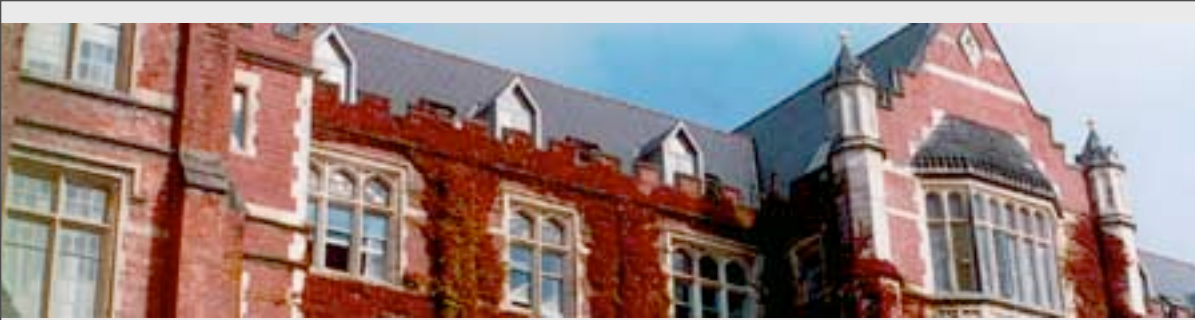
$$d_L(z) = d_H z \left\{ 1 - \frac{1}{2} [-1 + q_0] z + \frac{1}{6} [q_0 + 3q_0^2 - (j_0 + \Omega_0)] z^2 + O(z^3) \right\}.$$

$$d_F(z) = d_H z \left\{ 1 - \frac{1}{2} q_0 z + \frac{1}{24} [3 + 10q_0 + 12q_0^2 - 4(j_0 + \Omega_0)] z^2 + O(z^3) \right\}.$$

$$d_P(z) = d_H z \left\{ 1 - \frac{1}{2} [1 + q_0] z + \frac{1}{6} [3 + 4q_0 + 3q_0^2 - (j_0 + \Omega_0)] z^2 + O(z^3) \right\}.$$

$$d_Q(z) = d_H z \left\{ 1 - \frac{1}{2} [2 + q_0] z + \frac{1}{24} [27 + 22q_0 + 12q_0^2 - 4(j_0 + \Omega_0)] z^2 + O(z^3) \right\}.$$

$$d_A(z) = d_H z \left\{ 1 - \frac{1}{2} [3 + q_0] z + \frac{1}{6} [12 + 7q_0 + 3q_0^2 - (j_0 + \Omega_0)] z^2 + O(z^3) \right\}.$$



Pick a specific  
Hubble law:

$$d_F(z) = d_H z \left\{ 1 - \frac{1}{2} q_0 z + \frac{1}{24} [3 + 10q_0 + 12q_0^2 - 4(j_0 + \Omega_0)] z^2 + O(z^3) \right\}.$$

Transform it:

$$\begin{aligned} \ln[d_F/(z \text{ Mpc})] &= \frac{\ln 10}{5} [\mu_D - 25] - \ln z - \frac{1}{2} \ln(1 + z) \\ &= \ln(d_H/\text{Mpc}) \\ &\quad - \frac{1}{2} q_0 z + \frac{1}{24} [3 + 10q_0 + 9q_0^2 - 4(j_0 + \Omega_0)] z^2 + O(z^3). \end{aligned}$$

- simple probe for deceleration parameter
- stellar magnitude and redshift provided in the data
- plot the data...



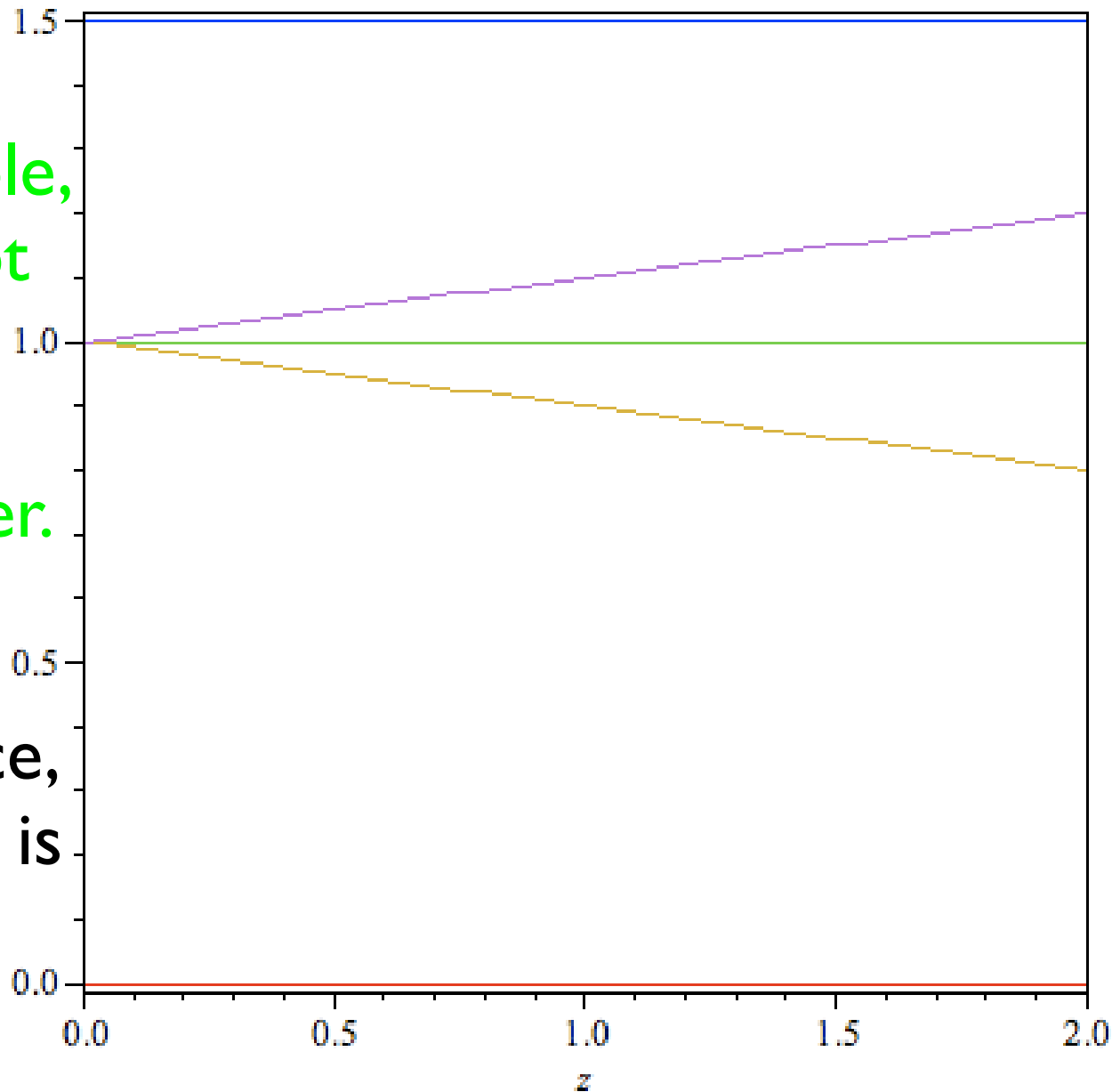


We expect something like this:



In principle, intercept yields Hubble parameter.

In practice, intercept is noise...



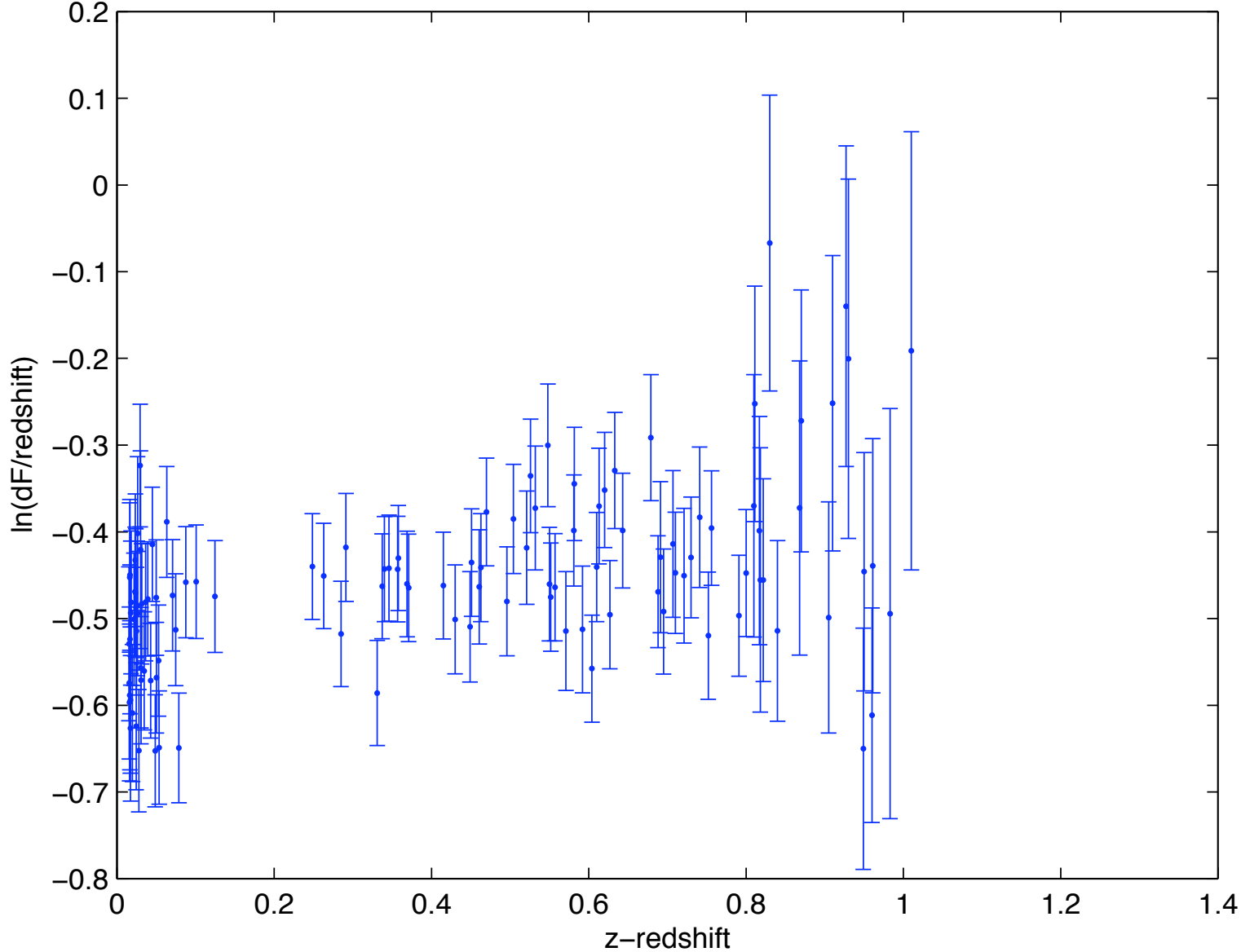
Slope yields:  
acceleration  
coasting  
deceleration

(overall calibration difficult...)

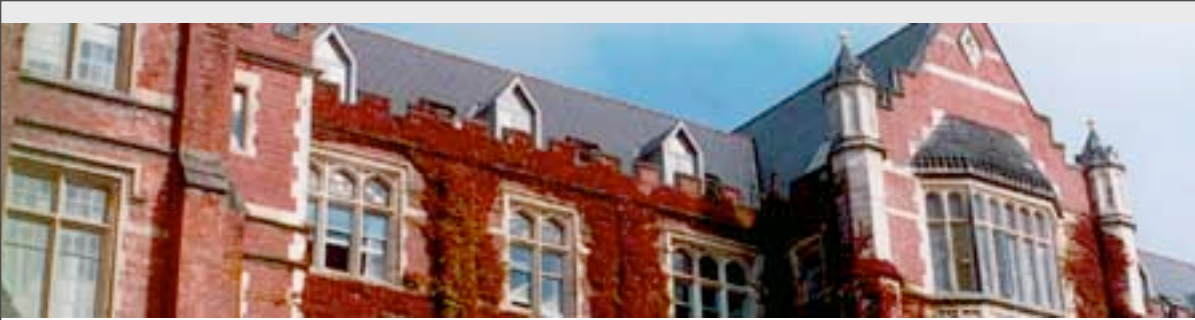


We get this:

Logarithmic Photon flux distance versus z-redshift using legacy05

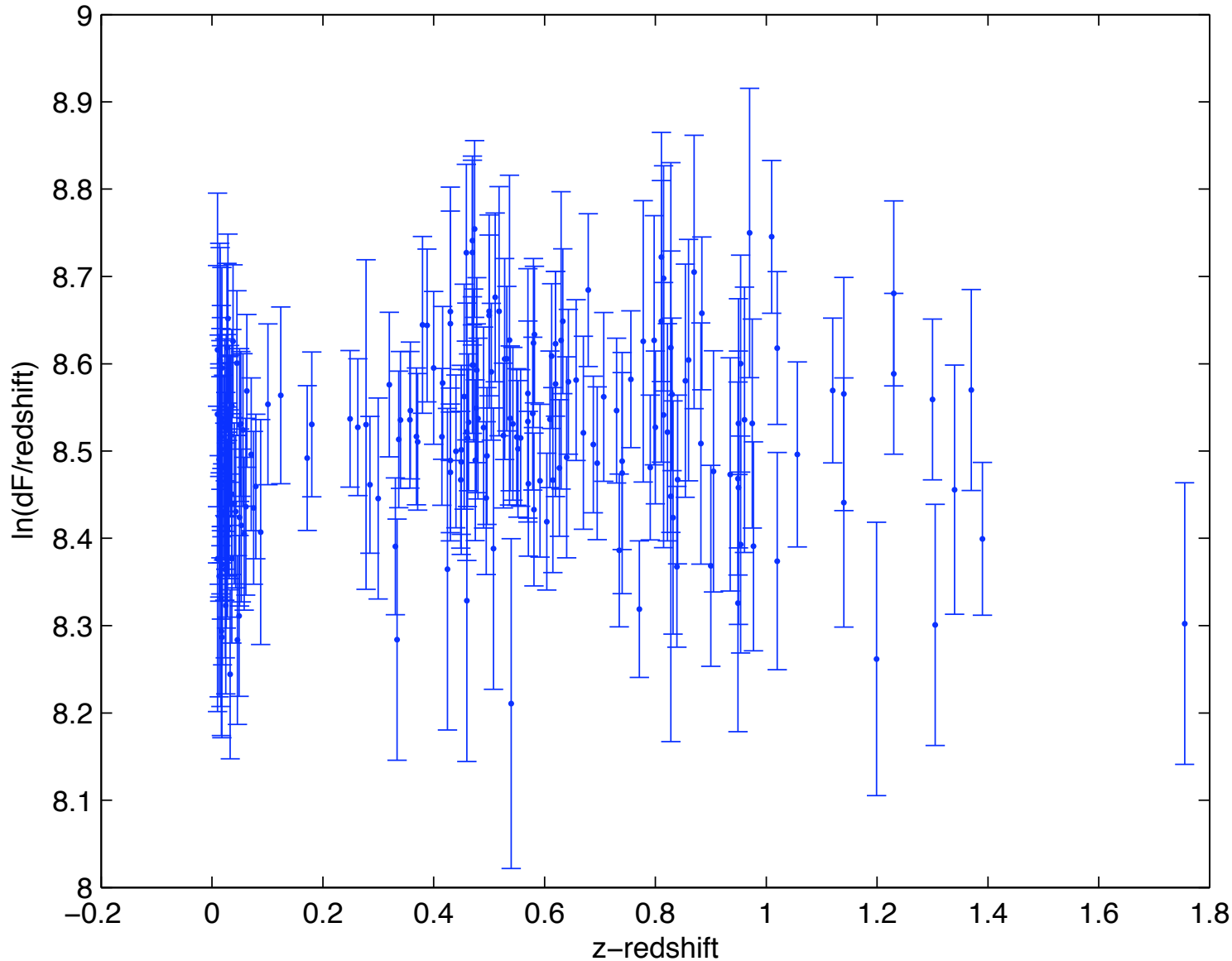


Smaller  
dataset,  
but  
homogeneous.



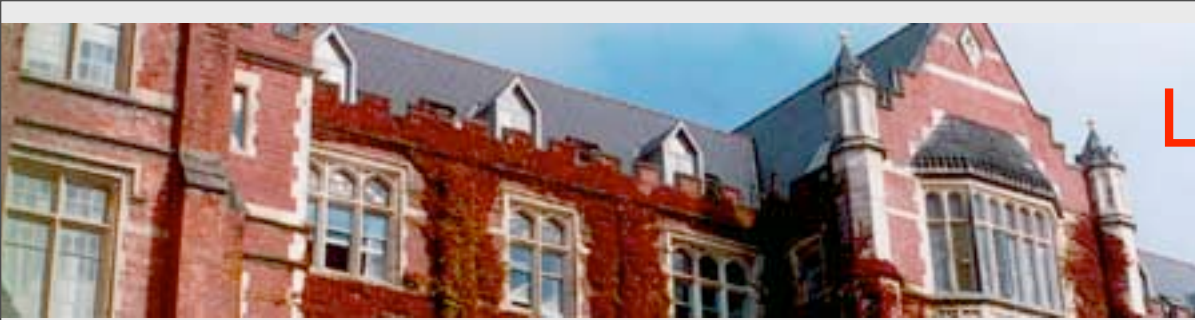
We get this:

Logarithmic Photon flux distance versus z-redshift using gold06



Larger  
dataset,  
but not  
homogeneous.

Combined  
dataset from  
six different  
observing  
platforms.



Lies, damned lies,  
and statistics...

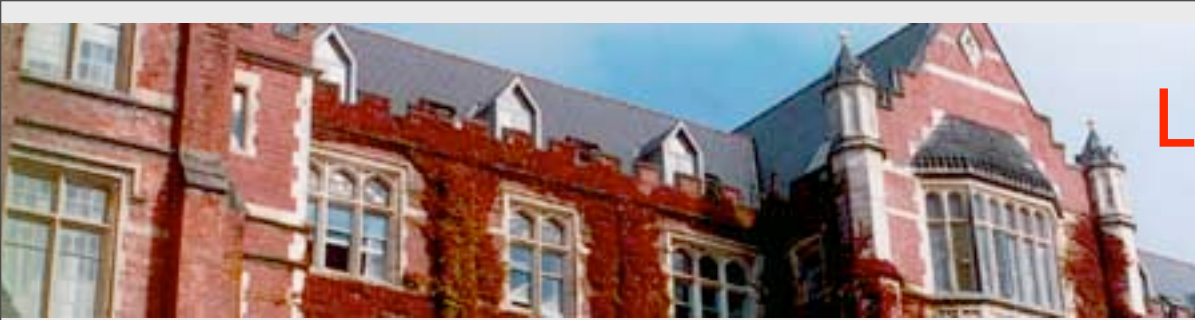


The situation is actually worse than it looks because the plotted error bars report only part of the uncertainty..

The plots include photometric uncertainties plus “intrinsic variability” in the supernovae...

The supernovae are not quite “standard candles”, they are only “standard on average”...

You have to estimate intrinsic variability by looking at nearby supernovae, where we have independent distance measurements...



Lies, damned lies,  
and statistics...

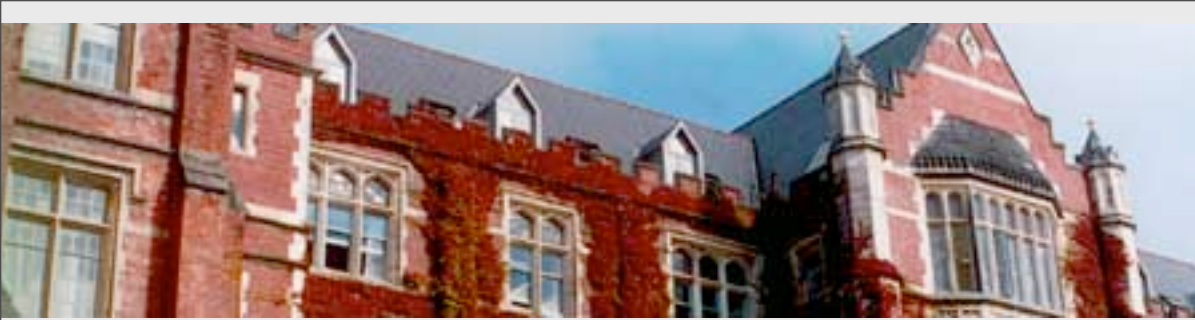


The plots do not include systematic uncertainties, neither “known unknowns” nor a budget for “unknown unknowns”.

(This is traditional in cosmology...)

“Known unknowns” are estimated to permit an uncertainty amounting to a drift of about 5% in distance measurements over a redshift range of:  $\Delta z = 1$ .

“Unknown unknowns” can be estimated historically...



## Historical uncertainties:

### Most recent:

- \* As of 2006 the high redshift supernovae have all moved 5% closer than estimated in 2004.  
(Improved understanding and characterization of nonlinearities in the photodetectors.)
- \* Over the last decade there have still been 15% disagreements over the size of our own galaxy...  
(Hipparcos satellite data.)
- \* Hubble's mis-calibrated Cepheid variables led to some 666% error...



## NIST guidelines:

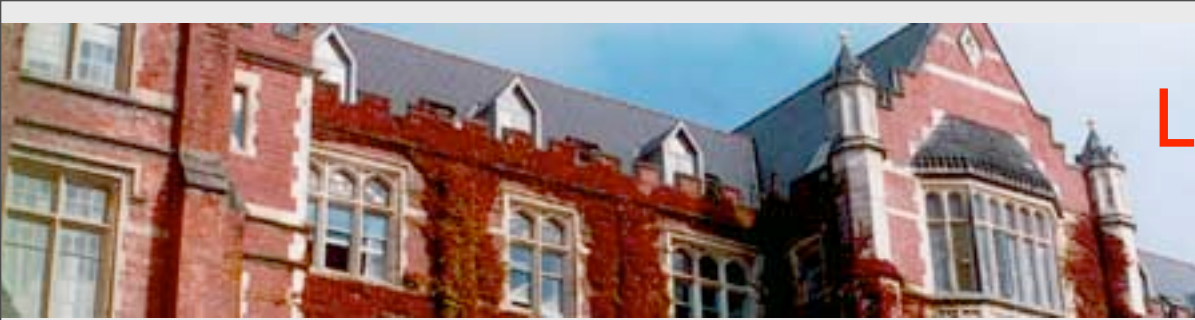


### Type B evaluations of uncertainty:

“any method of evaluation of uncertainty by means other than the statistical analysis of a series of observations”

“A type B evaluation of standard uncertainty is usually based on scientific judgment using all of the relevant information available, which may include: previous measurement data, etc...”

NIST Technical Note 1297.



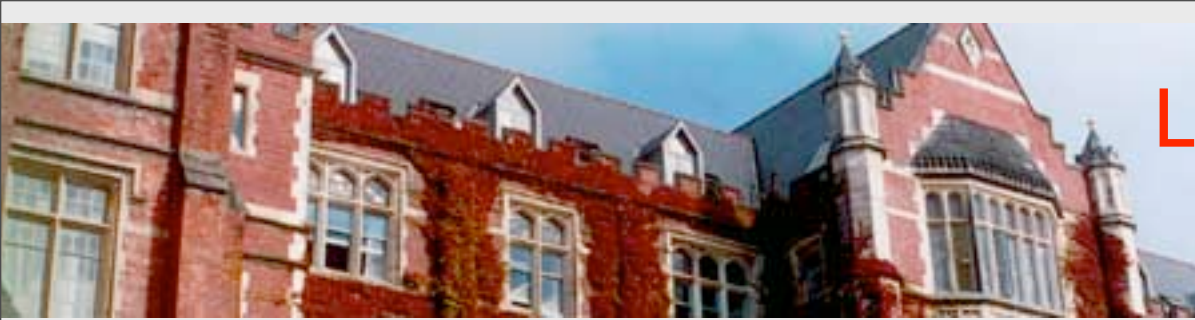
Lies, damned lies,  
and statistics...

In the total error budget you should really include:

statistical	photometric
	intrinsic
	* modelling *
systematic	known unknowns
	unknown unknowns

\* more on this later...

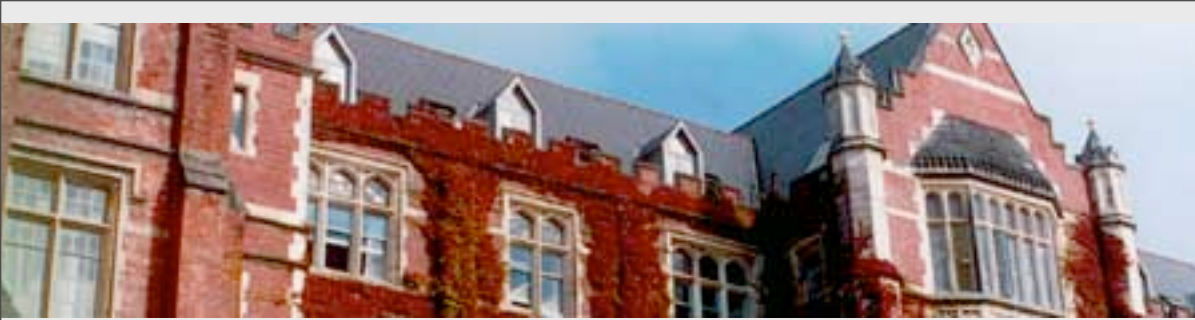




Lies, damned lies,  
and statistics...

## NIST recommended practice:

- \* Treat all uncertainties, whatever their source, “as though” they were statistical, and report an “equivalent one-sigma uncertainty”...
- \* Always combine uncertainties in quadrature, unless you have good reason to believe there is a correlation...
- \* In particular, combine statistical and systematic uncertainties in quadrature...



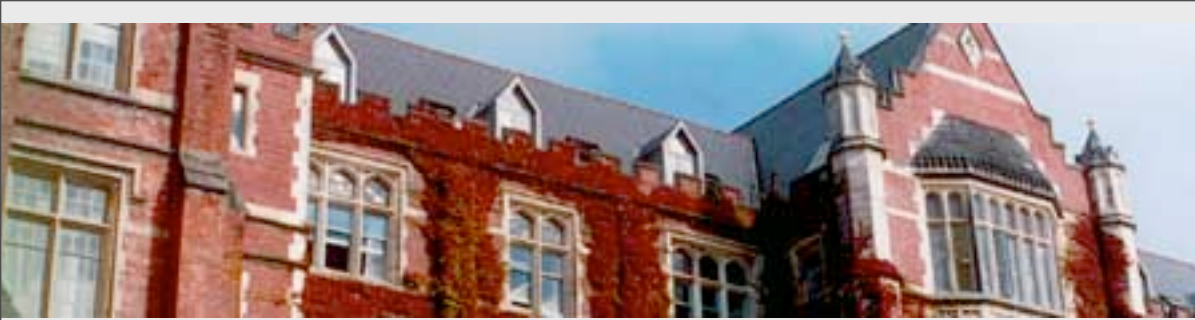
## Modelling uncertainty:

Essentially, ask the same statistical question several slightly different ways, and see if the estimates are close to each other...

On the next two slides are five different ways of estimating the deceleration and jerk parameters...

Perform least squares fits to the following five polynomials, all slightly different...

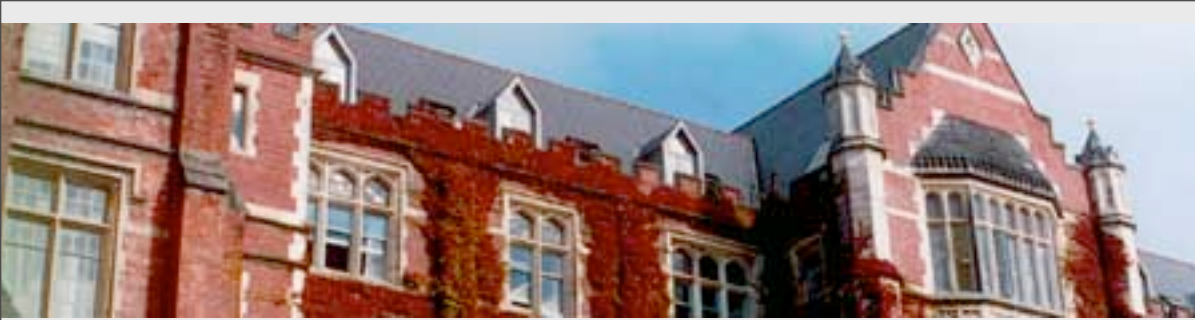
Then extract five (slightly?) different estimates of the deceleration and jerk parameters...



$$\begin{aligned}\ln[d_L/(z \text{ Mpc})] &= \frac{\ln 10}{5}[\mu_D - 25] - \ln z \\ &= \ln(d_H/\text{Mpc}) \\ &\quad - \frac{1}{2}[-1 + q_0]z + \frac{1}{24}[-3 + 10q_0 + 9q_0^2 - 4(j_0 + \Omega_0)]z^2 + O(z^3).\end{aligned}$$

$$\begin{aligned}\ln[d_P/(z \text{ Mpc})] &= \frac{\ln 10}{5}[\mu_D - 25] - \ln z - \ln(1 + z) \\ &= \ln(d_H/\text{Mpc}) \\ &\quad - \frac{1}{2}[1 + q_0]z + \frac{1}{24}[9 + 10q_0 + 9q_0^2 - 4(j_0 + \Omega_0)]z^2 + O(z^3).\end{aligned}$$

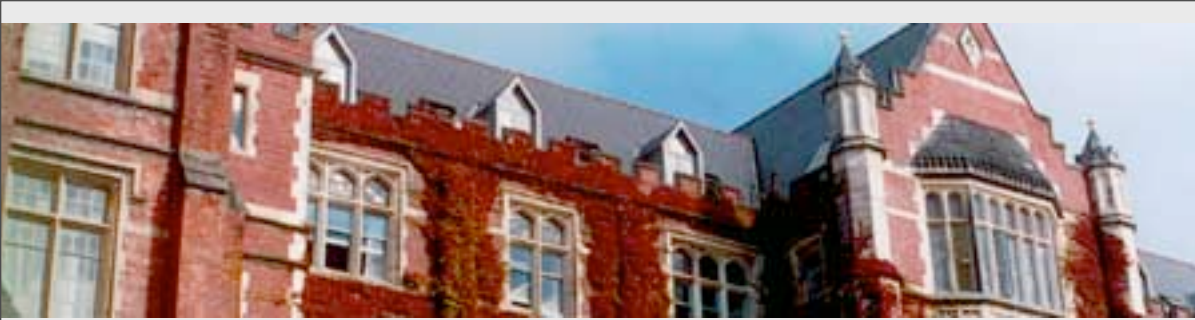
$$\begin{aligned}\ln[d_A/(z \text{ Mpc})] &= \frac{\ln 10}{5}[\mu_D - 25] - \ln z - 2 \ln(1 + z) \\ &= \ln(d_H/\text{Mpc}) \\ &\quad - \frac{1}{2}[3 + q_0]z + \frac{1}{24}[21 + 10q_0 + 9q_0^2 - 4(j_0 + \Omega_0)]z^2 + O(z^3).\end{aligned}$$



$$\begin{aligned}\ln[d_F/(z \text{ Mpc})] &= \frac{\ln 10}{5}[\mu_D - 25] - \ln z - \frac{1}{2} \ln(1 + z) \\ &= \ln(d_H/\text{Mpc}) \\ &\quad - \frac{1}{2}q_0 z + \frac{1}{24} [3 + 10q_0 + 9q_0^2 - 4(j_0 + \Omega_0)] z^2 + O(z^3).\end{aligned}$$

$$\begin{aligned}\ln[d_Q/(z \text{ Mpc})] &= \frac{\ln 10}{5}[\mu_D - 25] - \ln z - \frac{3}{2} \ln(1 + z) \\ &= \ln(d_H/\text{Mpc}) \\ &\quad - \frac{1}{2} [2 + q_0] z + \frac{1}{24} [15 + 10q_0 + 9q_0^2 - 4(j_0 + \Omega_0)] z^2 + O(z^3).\end{aligned}$$

**If the fit is robust, then these five analyses  
should give similar results...**



Huh, why are  
they different?

Because the process of performing a least squares fit  
**does not commute**  
with the process of truncating a Taylor series...

(And the amount by which these processes  
fail to commute gives you an estimate of the  
extent to which you should trust the  
output of the statistical analysis...)

((Trust me, you really do not want to  
see the relevant formulae))

[[Cattoen, Visser, gr-qc/0703122](#)]



# legacy05 dataset



distance	$q_0$	$j_0 + \Omega_0$
$d_L$	$-0.48 \pm 0.17$	$+0.43 \pm 0.60$
$d_F$	$-0.56 \pm 0.17$	$+1.16 \pm 0.65$
$d_P$	$-0.62 \pm 0.17$	$+1.92 \pm 0.69$
$d_Q$	$-0.69 \pm 0.17$	$+2.69 \pm 0.74$
$d_A$	$-0.75 \pm 0.17$	$+3.49 \pm 0.79$

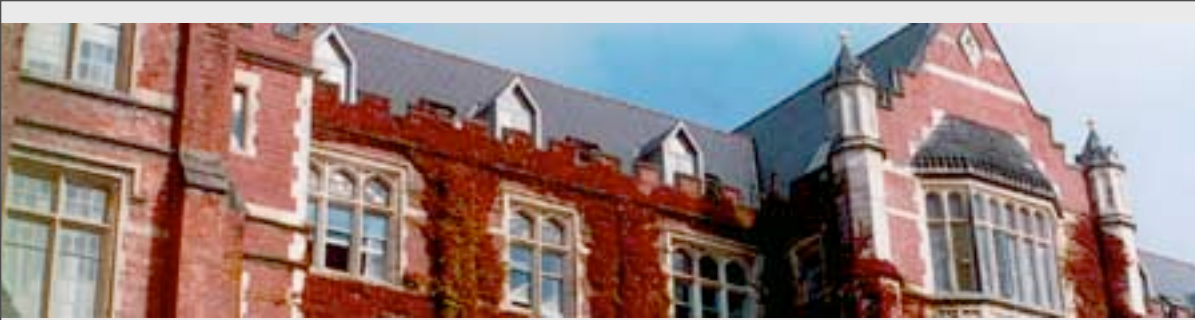
With  $1-\sigma$  statistical uncertainties.



gold06  
dataset

distance	$q_0$	$j_0 + \Omega_0$
$d_L$	$-0.37 \pm 0.11$	$+0.26 \pm 0.20$
$d_F$	$-0.48 \pm 0.11$	$+1.10 \pm 0.24$
$d_P$	$-0.58 \pm 0.11$	$+1.98 \pm 0.29$
$d_Q$	$-0.68 \pm 0.11$	$+2.92 \pm 0.37$
$d_A$	$-0.79 \pm 0.11$	$+3.90 \pm 0.39$

With  $1-\sigma$  statistical uncertainties.



y-redshift...

For good measure we then also transformed  
the redshift variable...

$$y = \frac{z}{1+z}; \quad z = \frac{y}{1-y}.$$

In the past (of an expanding universe)

$$z \in (0, \infty); \quad y \in (0, 1);$$

while in the future

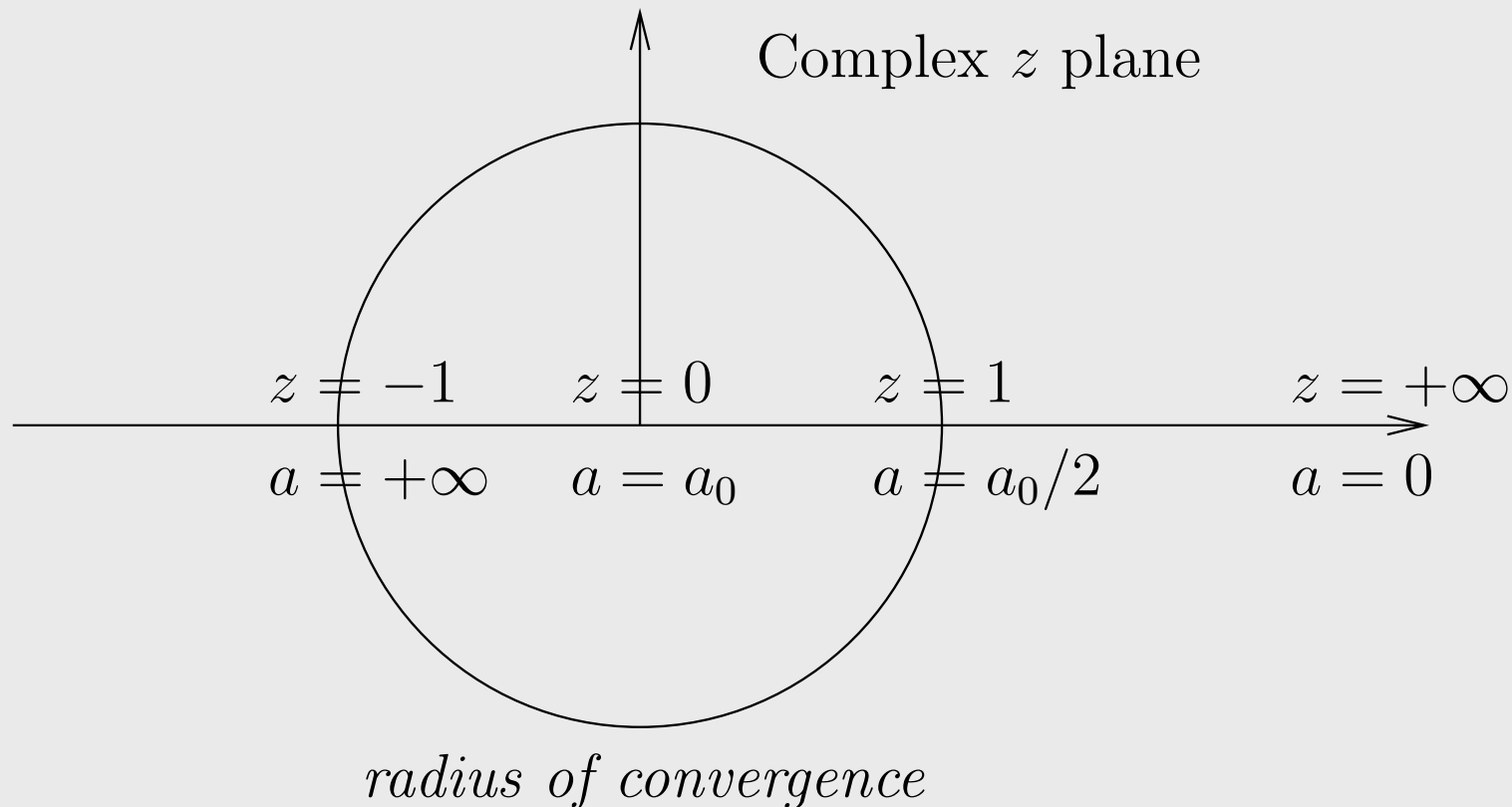
$$z \in (-1, 0); \quad y \in (-\infty, 0).$$





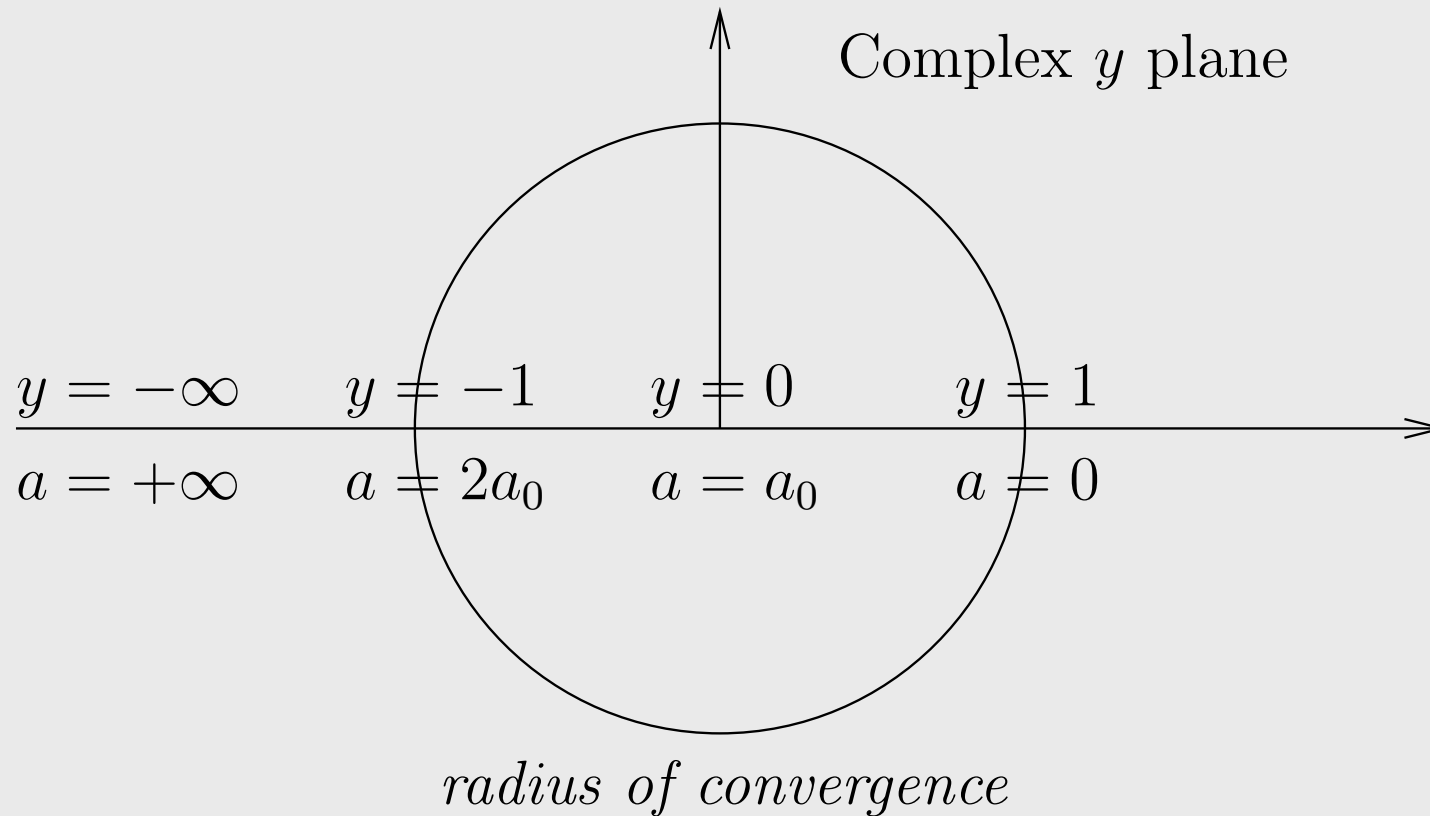
y-redshift...

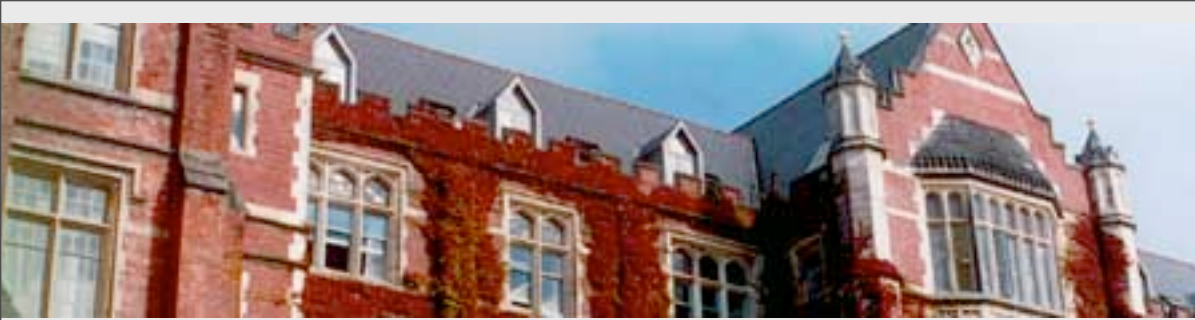
There are good theoretical reasons why y-redshift should be better behaved...





y-redshift...



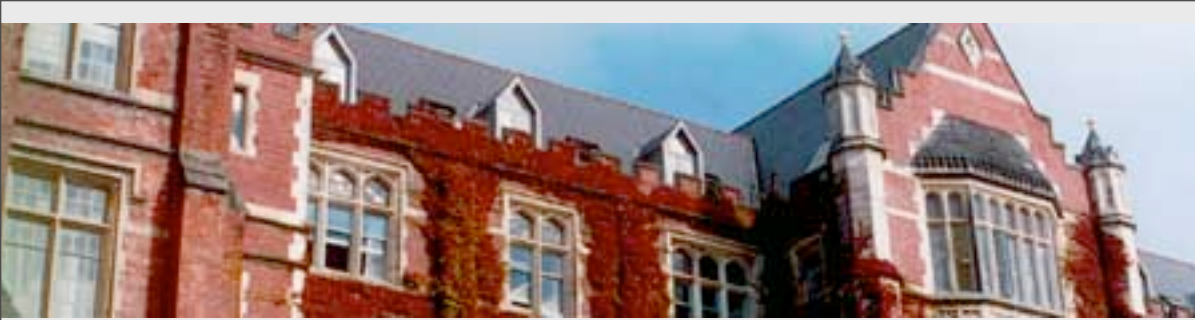


legacy05  
dataset  
y-redshift...



distance	$q_0$	$j_0 + \Omega_0$
$d_L$	$-0.47 \pm 0.38$	$-0.48 \pm 3.53$
$d_F$	$-0.57 \pm 0.38$	$+1.04 \pm 3.71$
$d_P$	$-0.66 \pm 0.38$	$+2.61 \pm 3.88$
$d_Q$	$-0.76 \pm 0.38$	$+4.22 \pm 4.04$
$d_A$	$-0.85 \pm 0.38$	$+5.88 \pm 4.20$

With  $1-\sigma$  statistical uncertainties.

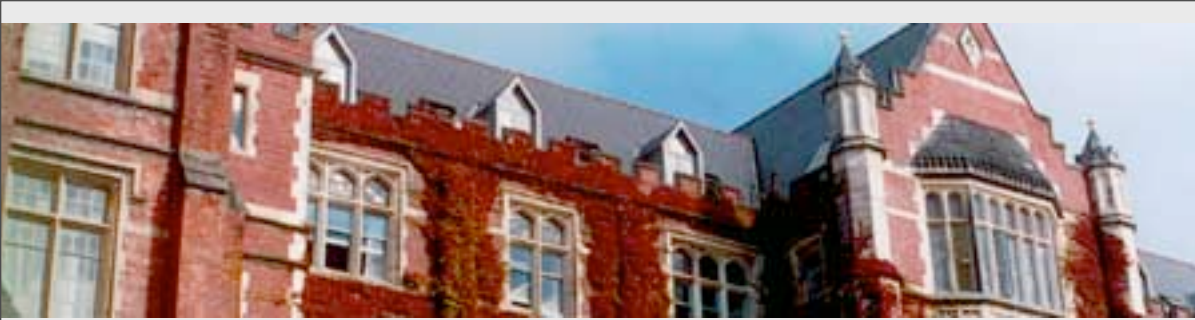


gold06  
dataset  
y-redshift...



distance	$q_0$	$j_0 + \Omega_0$
$d_L$	$-0.62 \pm 0.29$	$+1.66 \pm 2.60$
$d_F$	$-0.78 \pm 0.29$	$+3.95 \pm 2.80$
$d_P$	$-0.94 \pm 0.29$	$+6.35 \pm 3.00$
$d_Q$	$-1.09 \pm 0.29$	$+8.87 \pm 3.20$
$d_A$	$-1.25 \pm 0.29$	$+11.5 \pm 3.41$

With  $1-\sigma$  statistical uncertainties.



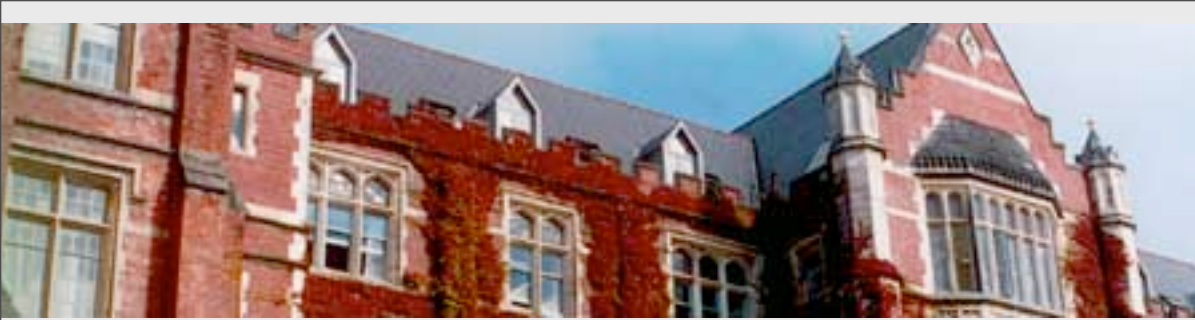
## Combine the analyses:



dataset	redshift	$q_0 \pm \sigma_{\text{statistical}} \pm \sigma_{\text{modelling}}$
legacy05	$y$	$-0.66 \pm 0.38 \pm 0.13$
legacy05	$z$	$-0.62 \pm 0.17 \pm 0.10$
gold06	$y$	$-0.94 \pm 0.29 \pm 0.22$
gold06	$z$	$-0.58 \pm 0.11 \pm 0.15$

With  $1-\sigma$  statistical uncertainties and  $1-\sigma$  model building uncertainties,  
no budget for “systematic” uncertainties.

(We shall draw a veil of discrete silence over the unfortunate status of the jerk parameter.)



Include  
systematics:

dataset	redshift	$q_0 \pm \sigma_{\text{statistical}} \pm \sigma_{\text{modelling}} \pm \sigma_{\text{systematic}} \pm \sigma_{\text{historical}}$
legacy05	$y$	$-0.66 \pm 0.38 \pm 0.13 \pm 0.09 \pm 0.09$
legacy05	$z$	$-0.62 \pm 0.17 \pm 0.10 \pm 0.09 \pm 0.09$
gold06	$y$	$-0.94 \pm 0.29 \pm 0.22 \pm 0.09 \pm 0.09$
gold06	$z$	$-0.58 \pm 0.11 \pm 0.15 \pm 0.09 \pm 0.09$

With  $1\text{-}\sigma$  effective statistical uncertainties for all components.

I think you can see where this is headed...

(Some astrophysicists think we should provide even larger historical uncertainties.)



## Combine uncertainties:

$$\sigma_{\text{combined}} = \sqrt{\sigma_{\text{statistical}}^2 + \sigma_{\text{modelling}}^2 + \sigma_{\text{systematic}}^2 + \sigma_{\text{historical}}^2}.$$

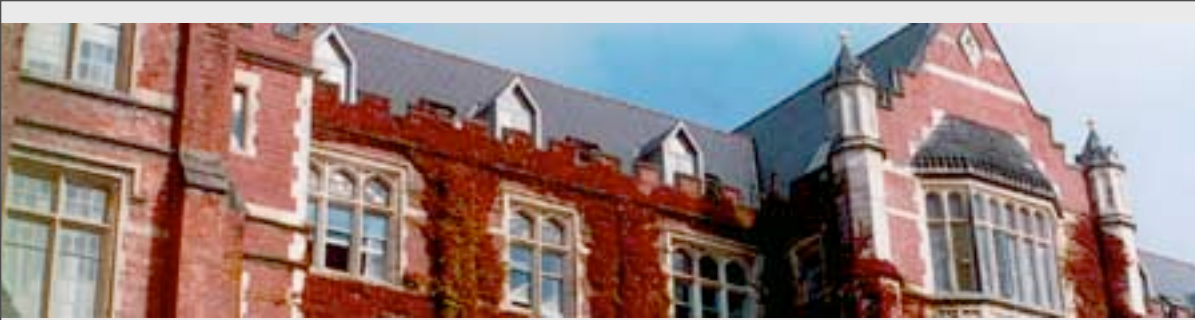
**Expanded uncertainty:**  $U_k = k \sigma_{\text{combined}}.$

Used when you need to be “certain” for either scientific or legal/ regulatory reasons...

Bitter experience in particle physics:

“If it’s not 3-sigma, it’s not physics...”

$$U_3 = 3 \sigma_{\text{combined}}.$$



## The 3-sigma standard:



Three-sigma corresponds to being 99.5% statistically sure you have a real effect...

Three-sigma is the minimum standard considered acceptable in particle physics before claiming “new physics”...

(This is of course a scientific judgment based on the historical record of what has worked in the past...)

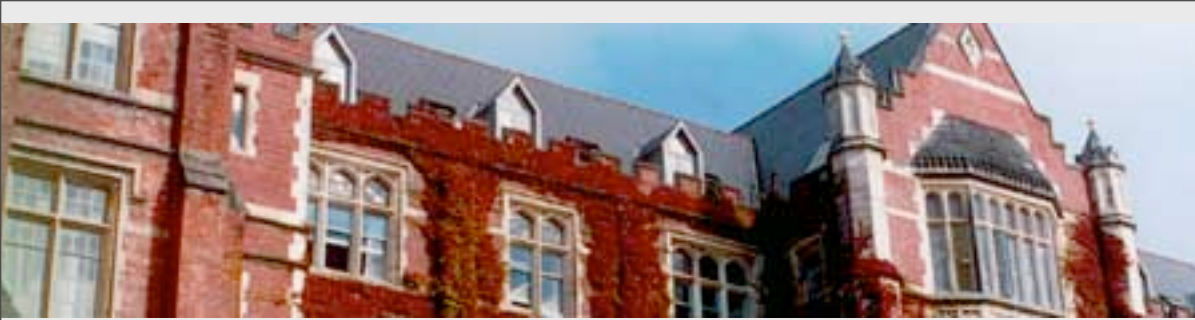




## The 3-sigma standard:

dataset	redshift	$q_0 \pm \sigma_{\text{combined}}$	$q_0 \pm U_3$
legacy05	$y$	$-0.66 \pm 0.42$	$-0.66 \pm 1.26$
legacy05	$z$	$-0.62 \pm 0.23$	$-0.62 \pm 0.70$
gold06	$y$	$-0.94 \pm 0.39$	$-0.94 \pm 1.16$
gold06	$z$	$-0.58 \pm 0.23$	$-0.58 \pm 0.68$

That is: **not statistically significant at three-sigma.**



Preponderance of evidence:

The universe is accelerating.

But (based on supernova data alone),  
this acceleration is not established  
“beyond reasonable doubt”.

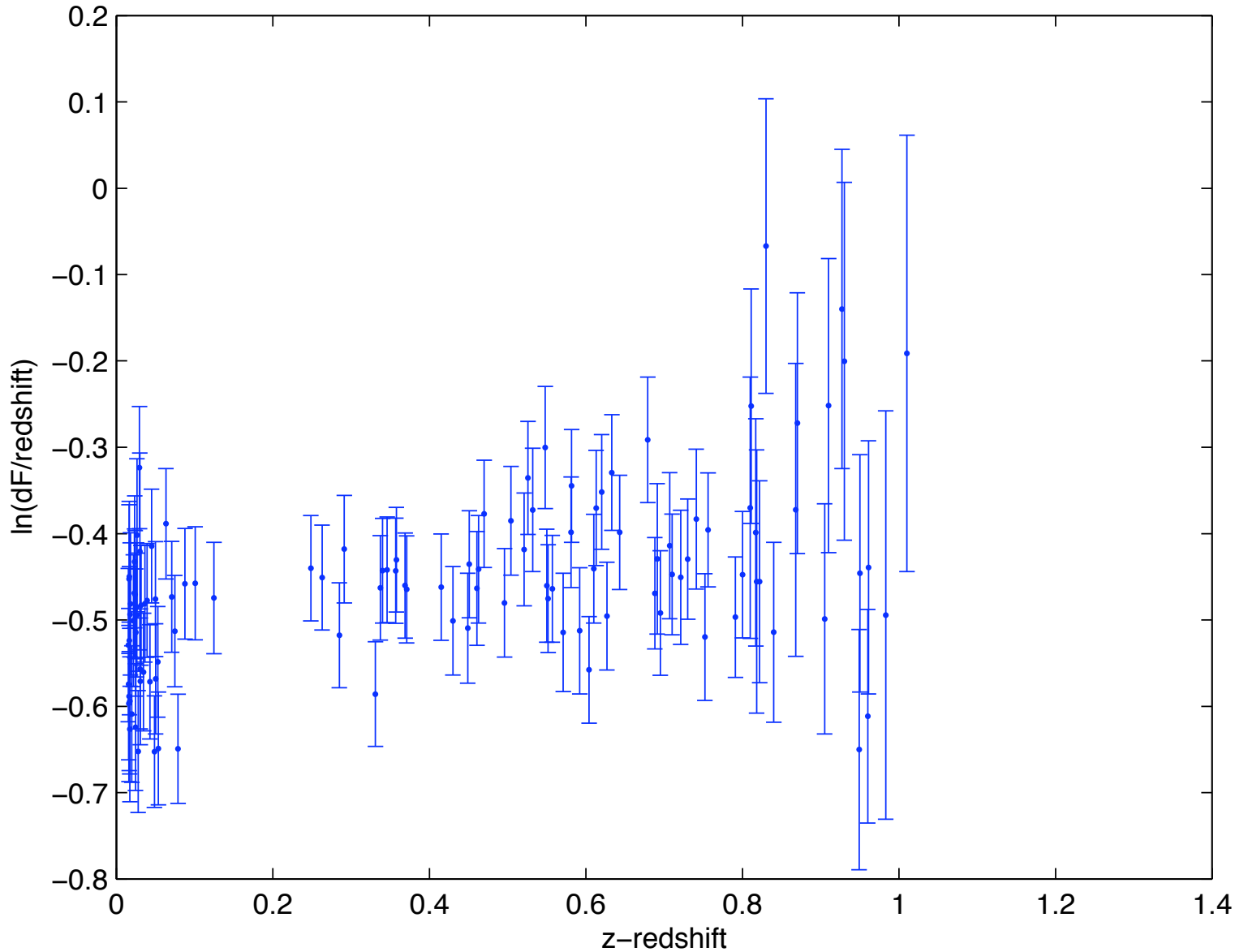
There are an awful lot of subtleties hiding in the  
woodwork of the statistical analyses...

Antidote to excessive statistical sophistication:



Antidote:

Logarithmic Photon flux distance versus z-redshift using legacy05



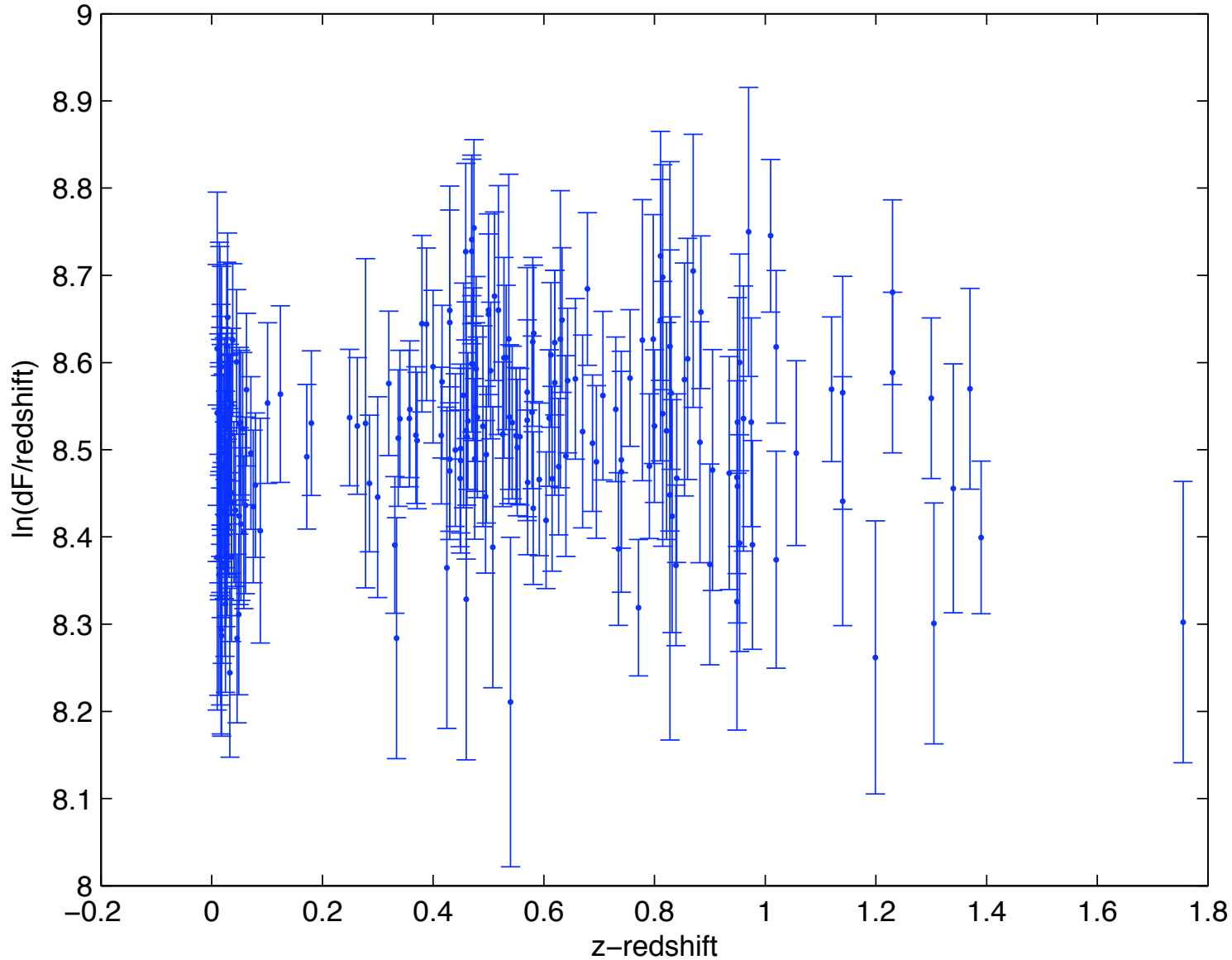
(statistical  
uncertainties  
only)



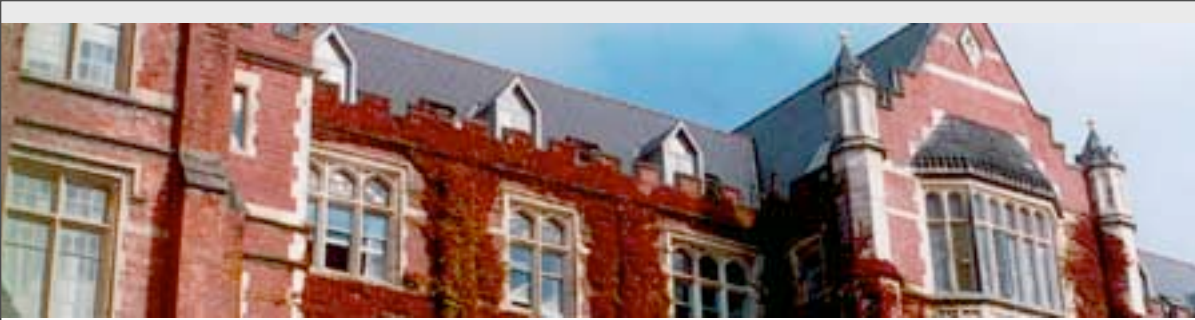
Antidote:



Logarithmic Photon flux distance versus z-redshift using gold06

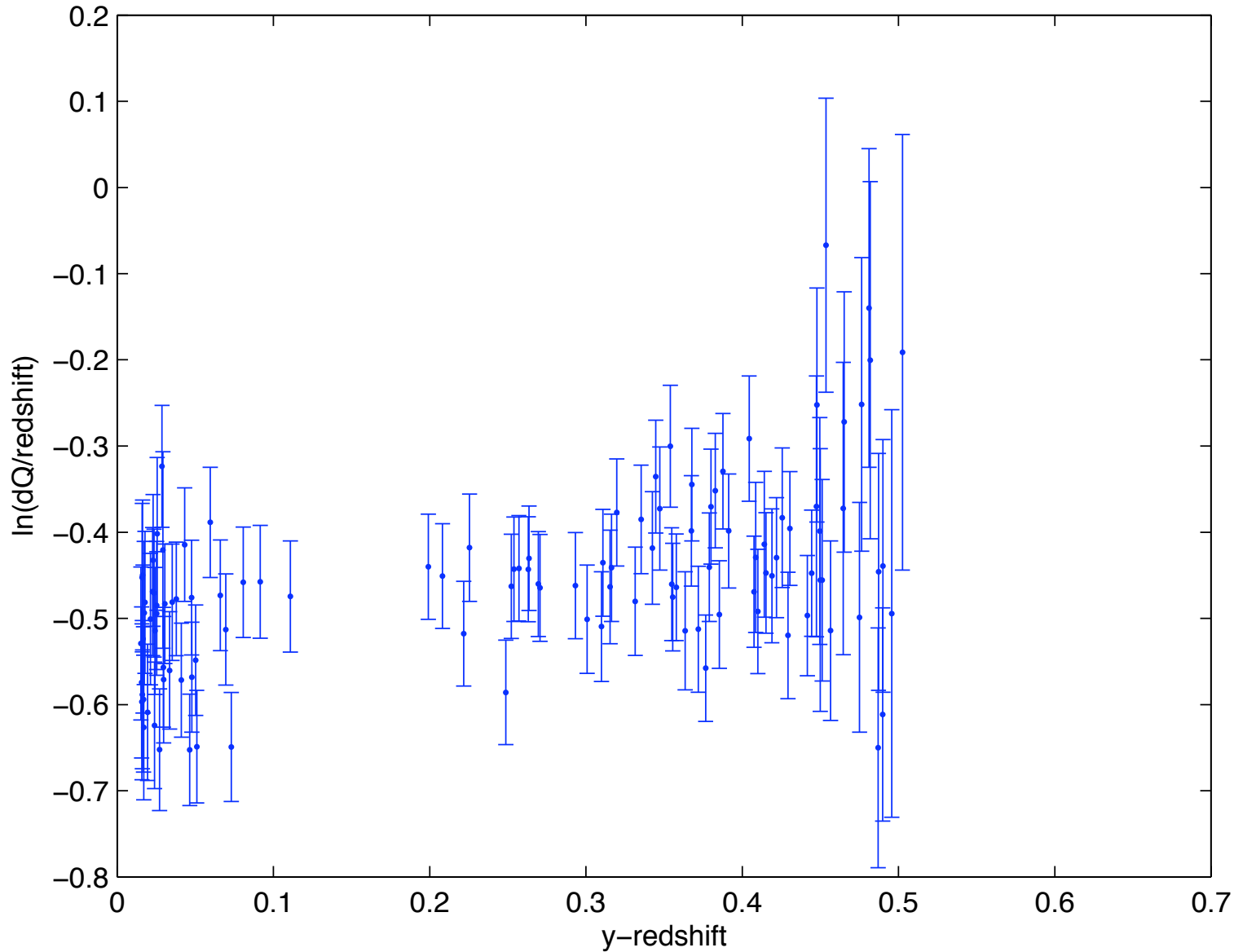


(statistical  
uncertainties  
only)

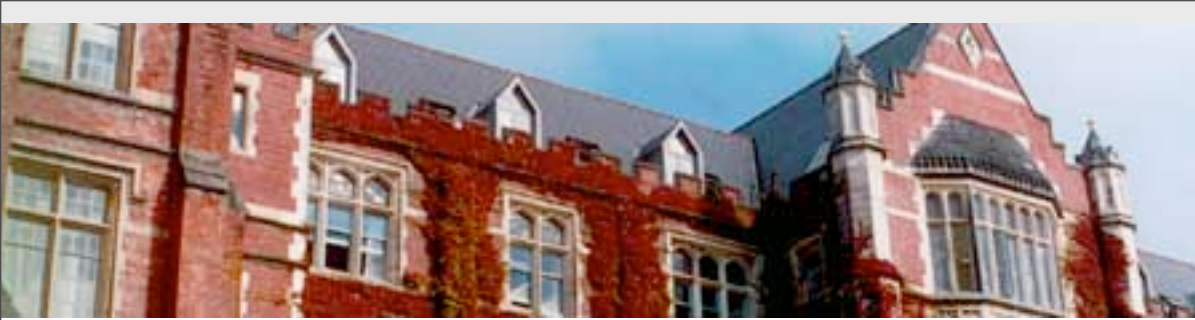


Antidote:

Logarithmic Deceleration distance versus  $y$ -redshift using legacy05

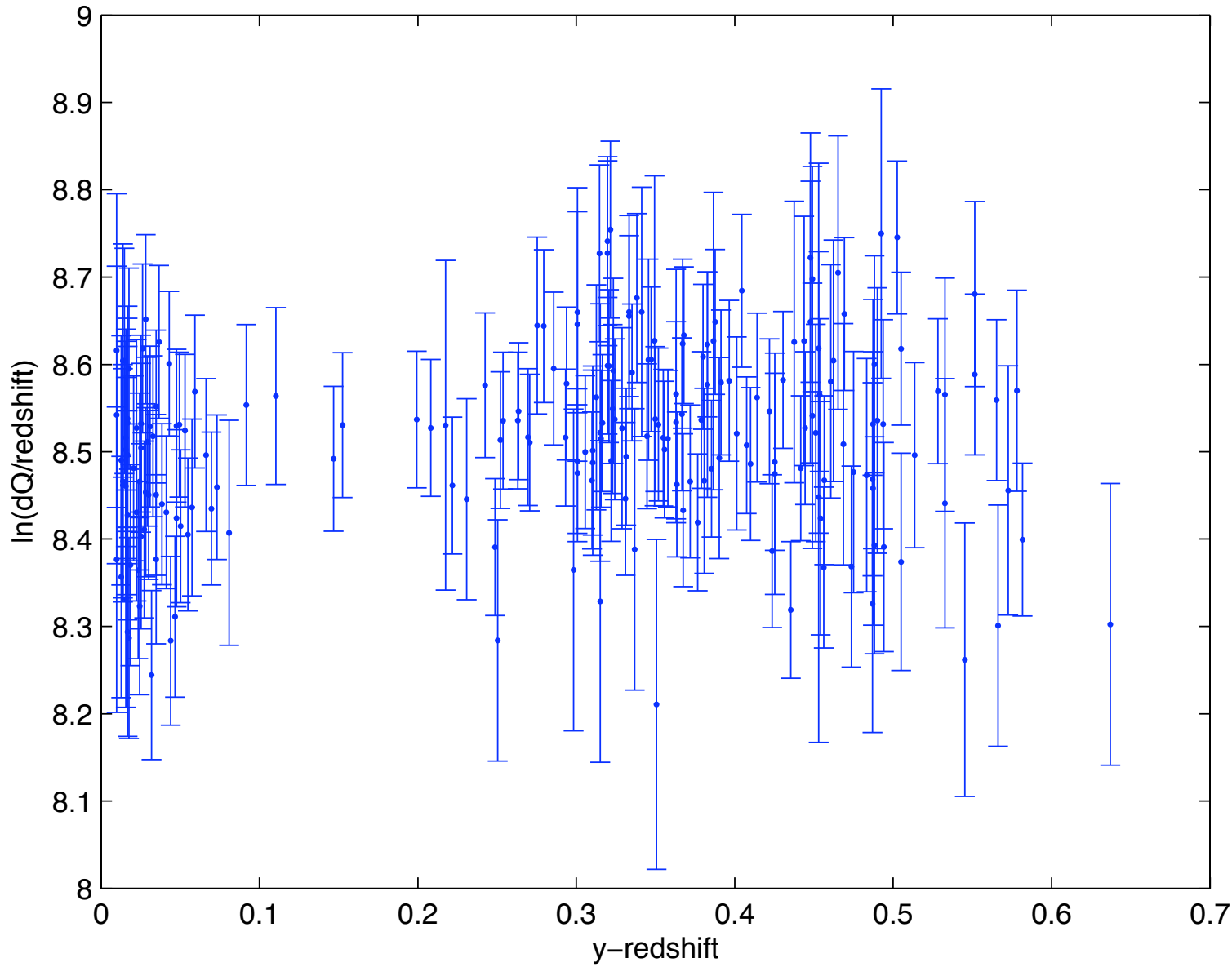


(statistical  
uncertainties  
only)



Antidote:

Logarithmic Deceleration distance versus  $y$ -redshift using gold06



(statistical  
uncertainties  
only)

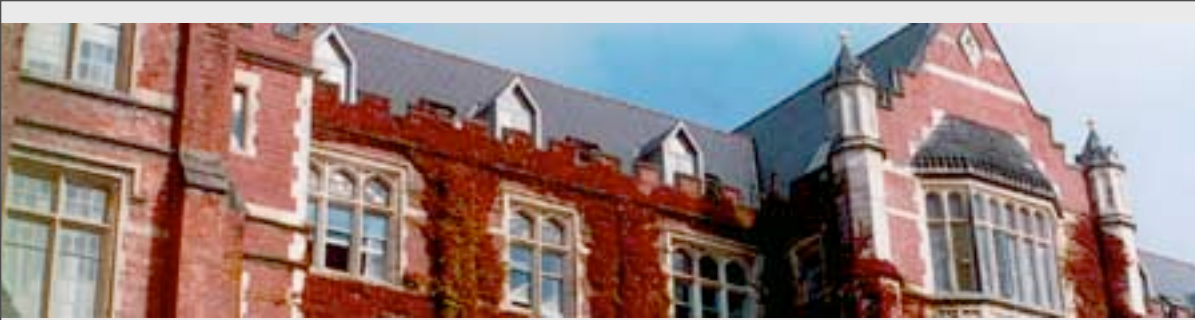


\* The fact that there is no overwhelmingly obvious visual trend in these four graphs tells you that extracting the deceleration parameter will at best be a very tricky and uncertain process.

\* However, the leading term in the Hubble law,

$$d = \frac{c z}{H_0} + \mathcal{O}(z^2),$$

is certainly well supported by the supernova data.



- \* Some parts of cosmology are already precision science.
- \* Cosmological distance determinations, however, are not yet precision science.

“Precision cosmology? Not just yet.”





“It is important to keep an open mind; just not so open that your brains fall out”

--- **Albert Einstein**

